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Assessing Hybrid Time Series Models for Analyzing Currency in Circulation Volatility in Nigeria

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ABSTRACT

The volatility of currency in circulation in Nigeria poses significant challenges to the country's economic stability. This study contributes to the literature by identifying the most suitable hybrid time series model for capturing volatility in Nigeria's currency in circulation. The study utilizes secondary data obtained from the documented records of Central Bank of Nigeria, covering the period from 1960 to 2023. Stationarity checking was carried using ADF and KPSS tests. The presence of heteroscedasticity in the residuals was examined using Scatter Plots and Breusch-Pagan test. Two phase methods were applied in fitting the four different models. In the first method ARIMA (1, 1, 1) model was fitted. In the second method the residuals of the ARIMA (1, 1, 1) were extracted to fit GARCH, FIGARCH, EGARCH and TGARCH, which result to the four different hybrid models. ARIMA-GARCH (1, 1, 1) was found to be the best model with least AIC, Bayes, Shibata, and Hannan Quinn. The model was diagnosed using ARC-LM Test and Ljung-and Box test. The results of the forecast revealed that since the conditional standard deviations (0.9476, 0.9476, 0.9475) are close to each other, this indicates stable volatility, low volatility and similar volatility level in the currency in circulation for the next three years. Thus, there is need for continuous monitoring sigma values for potential changes, adjust strategies if volatility increases or decreases significantly and future research can use TAR-FIGARCH model for future forecasts of the volatility.

Keywords:
Currency in Circulation, Hybrid Model, ARIMA, FIGARCH, EGARCH, TGARCH

INTRODUCTION

The study explores better forecasting methods for currency in circulation volatility in Nigeria to enhance monetary transmission. Currency is a medium of exchange, unit of account, and store of value that facilitates economic transactions (Khan, 2017). Currency in circulation refers to the total value of physical currency, such as coins and banknotes that are in use within (Bank for International Settlement [BIS], 2020. Currency in circulation is an important component of a country's money supply and plays a crucial role in facilitating economic transactions.

The level of currency in circulation can have significant implications for the economy. An increase in currency in circulation can lead to an increase in aggregate demand, which can stimulate economic growth (Friedman, 1969). However, excessive growth in currency in circulation can lead to inflation, as more money chases a constant quantity of goods and services (Cagan, 1956). Furthermore, a decrease in currency in circulation can lead to a decrease in aggregated demand, which can lead to economic contraction.

Time series modelling is a crucial tool for analyzing and forecasting economic time series data (Box et al., 2015). Economic time series data often exhibit complex dynamics, such as non-stationarity, nonlinearity, and volatility clustering, which can be challenging to model using traditional statistical techniques (Tsay, 2005). The use of advanced hybrid models, such ARIMA-GARCH, can provide a more accurate representation of the data and can capture the complex dynamics that are often present in economic time series data.

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Traditional time series models, such as Autoregressive Integrated Moving Average (ARIMA) model, may not be sufficient to capture the complex dynamics of currency in circulation (Tsay, 2005). The use of advanced time series models, such as ARIMA-GARCH, can provide a more accurate the volatility clustering and leverage effects that are often present in financial time series (Bollerslev, 1986). Research on currency in circulation using ARIMA-GARCH, ARIMA-FIGARCH, ARIMA-EGARCH, and ARIMA-TGARCH models can provide valuable insights into the dynamics of currency in circulation and can inform monetary policy decisions.

Hybrid models consistently outperform standalone models across various predictive metrics such Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Square Error (MSE), etc by combining strengths of multiple algorithms (Bassiouni and Farouk, 2024). Moreover, they effectively capture both linear and nonlinear patterns that single models alone often miss, improving performance on time series with mixed characteristics (Cao et al., 2025). Furthermore, hybrid models are more resilient to noise, outliers, and data irregularities, leading to more stable forecasts and stronger generalization on unseen data.

Studies on currency in circulation in Nigeria were carried out these include Ardo and Olorunfemi (2022) conducted a study on predictive modeling of Nigeria's currency in circulation using X-12 Autoregressive Integrated Moving Average (X-12 ARIMA) method. The study revealed that based on the data, it was clear that X-12-ARIMA (2 1 1)(0 1 1) was the most accurate forecasting approach for Nigeria's Currency in Circulation (CIC). The money in circulation in Nigeria from April 2022 through December 2022 will rise at a positive rate of 2.8% growth rate each month, with a predicted monthly mean CIC of 3.40 trillion by the end of the year 2022, according to this method's predictions. Furthermore, Atoyebi et al., (2023) employed Holt-Winters exponential smoothing method to forecast currency in circulation in Nigeria. The study aimed to investigate the forecasting of currencies in circulation (CIC) in Nigeria using the Holt-Winters exponential smoothing method (Additive Holt–Winters Model and Multiplicative Holt–Winters Method). The forecasting data were collected from January 1960 to December 2022. The study focused to determine the optimal forecasting approach while considering the pertinent smoothing parameters and determine which of the forecasting method, either the multiplicative Holt–Winters or additive Holt–Winters was best in forecasting CIC in Nigeria. Based on the comparison of the accuracy measures, the study concluded that the multiplicative Holt-Winters method outperformed the additive Holt-Winters model in all three measures: Mean Absolute Percentage Error (MAPE), Mean Absolute Deviation (MAD), and Mean Square Deviation (MSD). The multiplicative Holt-Winters method has a significantly lower MAPE (5.97894E+00) compared to the additive Holt-Winters model (9.55758E+02). The forecast of CIC in Nigeria was conducted using the multiplicative Holt-Winters method, and it found that currency in circulation in Nigeria continues to increase as it shows the upper trend. In addition, Adubisi et al., (2017) investigated money in circulation in Nigeria using SARIMA model. Data on monthly records of money in circulation obtained from the central bank of Nigeria web database from January, 2000 to December, 2016 was analyzed using the Box-Jenkins (ARIMA) methodology. The Seasonal ARIMA (2, 1, 0) $(0, 1, 1)_{12}$ model was found to be appropriate in describing the patterns observed in the series. The model having passed the basic ARIMA diagnostic test was used to forecast for the next three years. The study result revealed that the money in circulation were rising steadily given the years considered. However, most of the reviewed studies in Nigerian context employed linear time series models to analyze currency in circulation. However, these models are ill-equipped to capture intricate economic time series characteristics, including volatility, long memory, asymmetry, heteroscedasticity, and structural breaks. In contrast, nonlinear hybrid time series models can effectively capture both linear and nonlinear characteristics of a time series, rending them more powerful forecasting tools than their linear counterparts. Notably, this research could not identify any existing studies that applied nonlinear time series models to forecast volatility in currency circulation in Nigeria. This study aims to fill that gap by evaluating and comparing the forecasting performance of four hybrid time series models (ARIMA-GARCH, ARIMA-FIGARCH, ARIMA-EGARCH, and ARIMA-TGARCH) in modelling and predicting the volatility of currency in circulation in Nigeria with following objectives; (i) To fit each hybrid model to the historical data on currency in circulation (ii) To assess and compare the model's abilities to capture trends and volatility clustering in the data (iii) To identify which hybrid model provides the best prediction and risk management insights for currency in circulation volatility in Nigeria (iv) To make some forecast of volatility of currency in circulation. Hence our focal research questions are: (i) Do autocorrelation and heteroscedasticity affect the behavior and characteristics of the economic time series (currency in circulation) data? (ii) Can the ARIMA model treat the autocorrelation and heteroscedasticity when incorporating with a GARCH, FIGARCH, EGARCH, and TGARCH models? (iii) Does the employed model reduce the volatility persistency in the residual of economic time series variable?

MATERIALS AND METHODS

Method of Data Collection

The data used for this research work are secondary data retrieved from the official website of the Central Bank of

Nigeria (www.c.b.n.gov.ng website). The data consists of economic time series values on currency in circulation in Nigeria. The data cover a period of 63 years (1960-2023) and are measured in billions of Nigerian Naira.

Method of Data Analysis

This section consists of the details of the methods employed for the data analysis in chapter four. The methods proposed for the data analysis in chapter four of the research work include: Time Series Plot, Augmented Dickey Fuller (ADF) Test, Autocorrelation Function (ACF) Plot, Partial Autocorrelation Function (PACF) Plot, Scatter Plot, GARCH, ARIMA-GARCH, ARIMA-FIGARCH, ARIMA-EGARCH, and ARIMA-Threshold GARCH, ARCH-LM, Breusch-Pagan Test, and Evaluation Metric Measures.

Time Visualization

A time Series plot was used to visualize the behavior of currency in circulation over the 63-years period. This helped in identifying long-term patterns, trends, and potential structural breaks in the data.

Augmented Dickey-Fuller Test

The study checked for unit roots in the time series with the test which was developed by Said and Dickey-Fuller (1984). The ADF test equation is given as:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \delta_1 \Delta Y_{t-1} \dots + \delta_p \Delta Y_{t-p} + \varepsilon_t \quad (1)$$

Where,

ΔY_t is the first difference of the time series Y_t, α is the intercept term, βt is the trend term, γ is the coefficient on the lagged level of the series Y_{t-1}, ε_t is the error term, p is the number of lagged differences included in the model, and $\delta_1, \dots, \delta_p$ are the coefficients on the lagged differences of the series $(\Delta Y_{t-1}, \dots, \Delta Y_{t-p})$.

The test involves the following hypotheses:

H_0: the time series has unit roots.

H_a: the time series non-unit roots.

Decision Criteria

Null hypothesis is rejected if P-value is less than the alpha value.

(Autocorrelation Function (ACF) Plot

This study employed ACF plot to identify the order of Moving Average (MA) model used in the constructed model and checked for presence of long memory. The function is defined as:

$$\rho(k) = \frac{\gamma(k)}{\gamma(0)} \quad (2)$$

$$\gamma(k) = \sum \frac{[(X_t - \mu)(X_{t+k} - \mu)]}{n-k} \quad (3)$$

Partial Autocorrelation (PACF) Plot

This research used Partial Autocorrelation Function (PACF) plot to identify the order of Autoregressive (AR) model used in the fitted models. The function is defined as:

$$\varphi(k) = \frac{|\tau(k)|}{|\tau(k-1)|} \quad (4)$$

Where,

k : is the number of lags.
$\tau(k)$: is the autocorrelation matrix up to lag k.

Scatter Plot

A scatter plot of the residuals over time was used to check for heteroscedasticity. The residuals were plotted on the vertical axis, and time was plotted on the horizontal axis. Evidence of heteroscedasticity was inferred from patterns in the spread of residuals.

Assumptions of the Models

i. Stationarity.
ii. Presence of conditional heteroscedasticity.
iii. Captures volatility.
iv. Absence of serial correlation in mean residuals.
v. Zero mean for residuals.
vi. Non-negativity of the parameters.

Software Employed

The study employed R software for the entire analysis conducted in chapter four, using the rugarch package.

ARIMA-GARCH Model

The ARIMA model captures mean, trend, and short memory of a time series but, does not account for, volatility clustering, or heteroscedasticity which are address by the GARCH model. The hybrid ARIMA-GARCH model is defined as:

$$\sigma_t^2 = \omega + \alpha_i \sum_{i=1}^{q} (z_t)_{t-i}^2 + \beta_j \sum_{j=1}^{p} \sigma_{t-j}^2 \quad (5)$$

Where, $\sigma^2{}_t$ is the conditional variance, z_t is the residual of ARIMA model, $\sigma^2{}_{t-j}$ is the previous volatility, ω is the intercept, α_i is the adjustment of the past shocks and β_j is the adjustment to the past volatility.

ARIMA-FIGARCH Model

The ARIMA-FIGARCH model addresses long memory in volatility, unlike standard GARCH. it is defined as:

$$\sigma_t^2 = \omega + \alpha_i \sum_{i=1}^{q} (z_t)_{t-i}^2 + \beta_j \sum_{j=1}^{p} \sigma_{t-j}^2 + [1 - (1 - L)^d] z_t^2 \quad (6)$$

Where, $\sigma^2{}_t$ is the conditional variance, z_t is the residual of the ARIMA model, $\sigma^2{}_{t-j}$ is the previous volatility, ω is the intercept, α_i is the adjustment of the past shocks and β_j is the adjustment to the past volatility, L is the lag operator, d is the differencing operator and $\varepsilon^2{}_t$ is the squared of the residuals.

ARIMA-EGARCH Model

EGARCH captures leverage effects and removes non-negativity constraints on the variance. The model is defined as:

$$log\sigma_t^2 = \omega + \beta log\sigma_{t-i}^2 + \alpha|\tau_{t-i}| + \gamma\tau_{t-i} \quad (7)$$

Where, $\log\left(\sigma^2{}_t\right)$ is the logarithm of the leverage effect which makes the leverage effect exponential rather than quadratic , this ensures that the estimates are non-negative, ω is the constant, α is the ARCH effect, τ_{t-i} is standardized residual of ARIMA model at time $t-i$ (previous shocks), γ is the asymmetric effects and β is the GARCH effects.

ARIMA-TGARCH Model

TGARCH introduces threshold asymmetry in volatility response. The model is defined as:

$$\sigma_t^2 = \omega + \alpha\tau_{t-i}^2 + \gamma\varepsilon_{t-i}^2 I(\tau_{t-i} < 0) + \beta\sigma_{t-i}^2 \quad (8)$$

ω is constant, α coefficient for previous squared shock, γ leverage term/asymmetry term (only adds if the residual of ARIMA model (τ) is negative, β coefficient for previous variance, $I(\tau_{t-i} < 0)$is indicator function (1 if previous shock is negative, 0 otherwise).

ARCH-LM Test

This study used the test proposed by Engle (1982) to check for the effect of heteroscedasticity in the residuals of the models. The test is defined as:

$$Q(m) = T(T+2)\sum_{j=1}^{m}\frac{\hat{\rho}_j}{T-L} \quad (9)$$

Where, m is the maximum numbers of lags included in the ARCH effect test, $\hat{\rho}_j$ is the sample Autocorrelation at lag j for the squared time series and T is the number of non-missing values in the data sample.

The test has the following hypotheses:

H_0: no Arch effect is present.

H_a: Arch effect is present.

We are to reject the null hypothesis if P-value is less than the alpha value.

Ljung-Box Test

The study checked for presence of serial correlation in the fitted models using a test proposed by Ljung-Box (1978). The test is defined as:

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{\tau_k^2}{n-k} \quad (10)$$

Where, n is the number of observations in the time series, k is the particular time lag to checked, m is the number of time lags to be tested, τ_k is the sample autocorrelation function of the k[th], residuals term.

The decision is based on the following hypotheses:

H_0: The residuals do not exhibit autocorrelation.

H_a: The residuals exhibit autocorrelation.

If (Q_m) value > critical value, reject the null hypothesis otherwise accept.

Evaluation Metric Test

This study used the following evaluation metric test to select the best model between the models fitted and to assess the forecast power of the selected best model.

i- Mean Absolute Error (MAE) =

$$\frac{1}{n}\sum_{t=1}^{n}\left|\sigma_t{}^2 - \hat{\sigma}_t{}^2\right| \quad (11)$$

ii- Mean Square Error (MSE) =

$$\frac{1}{n}\sum_{t=1}^{n}\left(\sigma_t{}^2 - \hat{\sigma}_t{}^2\right)^2 \quad (12)$$

iii- Root Mean Square Error (RMSE) =

$$\sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(\sigma_t{}^2 - \hat{\sigma}_t{}^2\right)^2} \quad (13)$$

iv- Akaike Information Criterion (AIC) =

$$2k - 2\ln(L) \quad (14)$$

Where, n is the sample size, σ_t^2 is the actual variance at time t and $\hat{\sigma}_t^2$ is the estimated variance at time t, K is the number of estimated parameters including intercept and L is the maximum likelihood of the model.

RESULTS AND DISCUSSION

Time Series Visualization

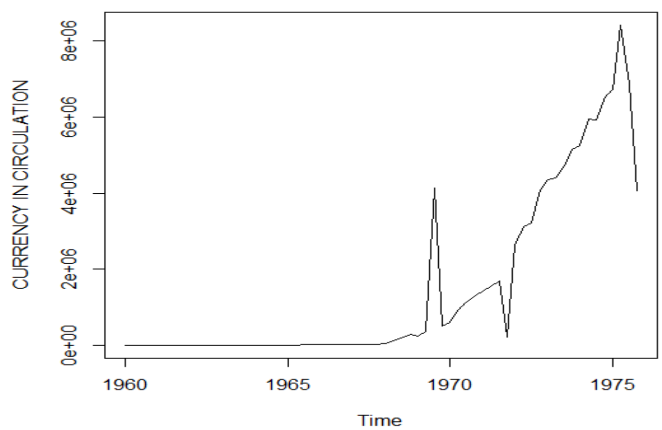

Figure 1 Time Series Visualization Plot

From the above result obtained in Figure 1 it has been observed that the behavior of the currency in circulation exhibits a trend with structural breaks.

Long Memory Checking

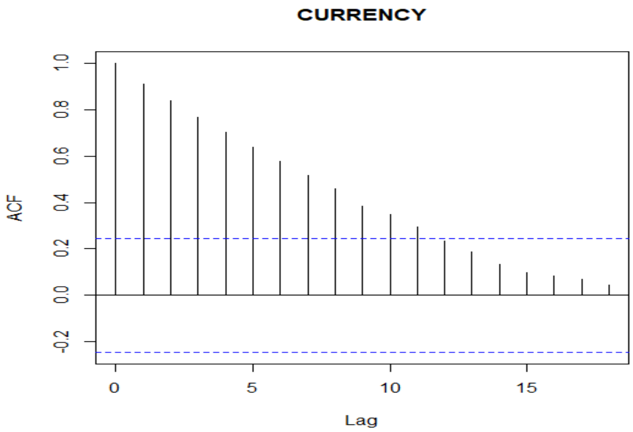

Figure 2 Autocorrelation Function Plot

From the result obtained in Figure 2 it is observed that the autocorrelations exhibit exponential decay. Thus, the currency in circulation has short memory, indicating the time series is not strongly affected by autocorrelation.

Table 1 ADF Test Results

Value of test-statistic is:	-1.0299	2.4234	2.3134
Critical values for test statistics:			
Values	1PCT	5PCT	10PCT
Tau3	-4.04	-3.45	-3.15

From the result obtained in Table 1 it is observed that the ADF test statistic is (-1.0299). This is greater than the alpha value 0.05 (-3.45). Thus, we fail to reject the null hypothesis and conclude that the currency in circulation time series is non-stationary.

Behavior Checking of the Differenced Time Series

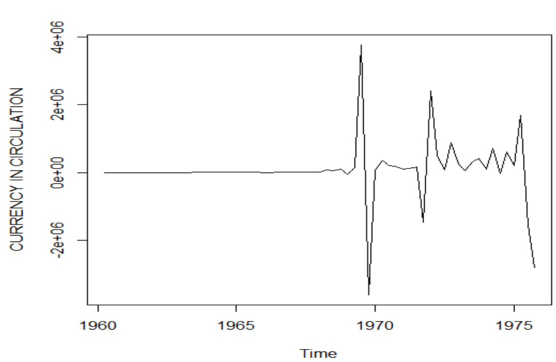

Figure 3 Differenced Time Series Plot

From Figure 3 it has been observed that the behavior of the currency in circulation after differencing appears random. Thus, the time series is now stationary.

Stationarity Checking of the Differenced Time Series

Table 2 ADF Test Results

Value of test-statistic is:	-3.9554	5.5156	8.2129
Critical values for test statistics:			
Values	1PCT	5PCT	10PCT
Tau3	-4.04	-3.45	-3.15

From the result in Table 2 it is observed that the ADF test statistic is (-3.9554). This is less than the alpha value 0.05 (-3.45). Thus, we reject the null hypothesis and conclude that the currency in circulation time series is stationary.

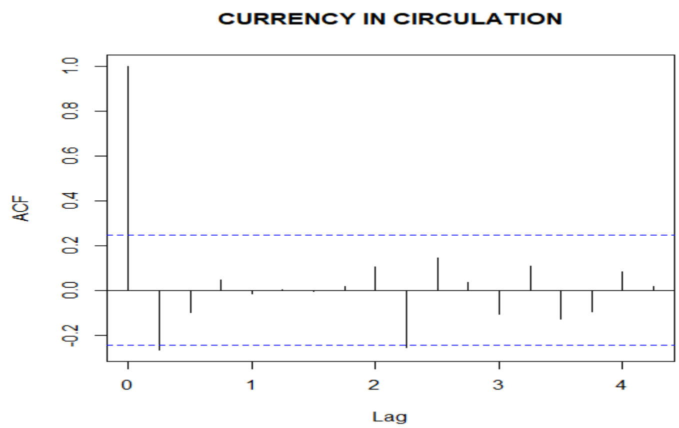

Figure 4 Autocorrelation Function Plot

From the results in Figure 4 it is observed that the autocorrelation plot shows significant spikes at lag 1, and lag 2. Thus, MA (1), and MA (2) are significant

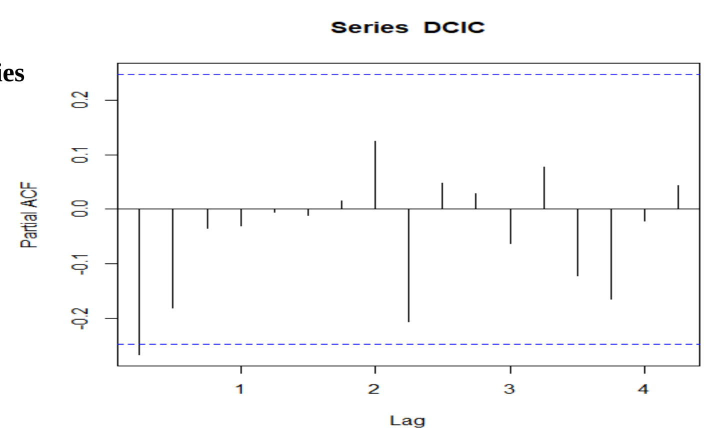

Figure 5 Partial Autocorrelation Function Plot

From the results in Figure 5 it is observed that the partial autocorrelation plot shows significant spikes at lag (1). Thus, AR (1) is significant.

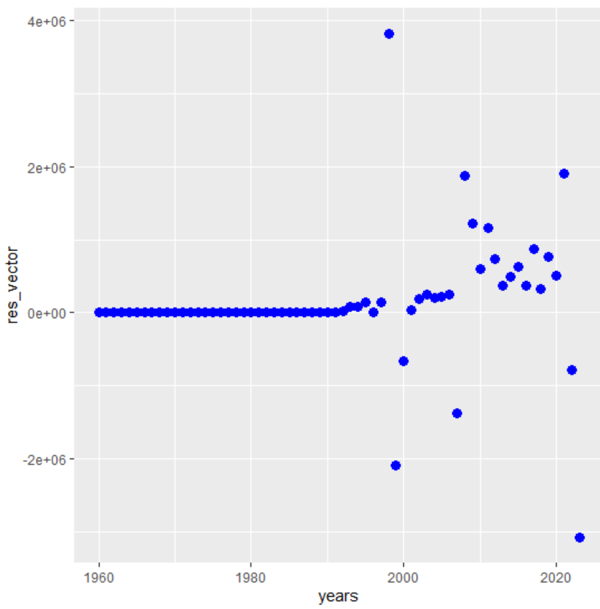

Figure 6 Scatter Plots of ARIMA Model Residuals

From the result in Figure 6 a fan pattern is observed. Thus, heteroscedasticity is present in the residuals, indicating that the time series is affected by heteroscedasticity.

Comparison of the Hybrids Models

Table 3 Information Criterion of the Models

Model	AIC
ARIMA (1, 1, 1)-GARCH (1, 1)	2.8240
ARIMA (1, 1, 1)-FIGARCH (1, 1)	2.8283
ARIMA (1, 1, 1)-EGARCH (1, 1)	2.8267
ARIMA (1, 1, 1)-TGARCH (1, 1)	2.8256

From the results in Table 3 it is observed that ARIMA (1, 1, 1)-GARCH (1, 1) is the best model with least AIC.

Table 4 Forecast Accuracy Measures of the Models

Model	MAE	MSE	RMSE
ARIMA (1, 1, 1)-GARCH (1, 1)	0.7812089	0.9798117	0.9898544
ARIMA (1, 1, 1)-FIGARCH (1, 1)	0.7812158	0.9798110	0.9898541
ARIMA (1, 1, 1)-EGARCH (1, 1)	0.7813938	0.9798120	0.9898547
ARIMA (1, 1, 1)-TGARCH (1, 1)	0.7812184	0.9798119	0.9898549

From the results in Table 4 it is observed that ARIMA (1, 1, 1)-GARCH (1, 1) is the best model with least MAE, MSE, and RMSE.

ARIMA (1, 1, 1)-GARCH (1, 1) Model

Table 5 Coefficients of ARIMA (1, 1, 1)-GARCH (1, 1)

Parameters	Estimate	Standard error	t value	Pr (>|t|)
μ	-0.043001	0.031253	-1.3759e+00	0.16886
ω	0.000850	0.000826	1.0293e+00	0.30332
α_1	0.000000	0.000916	4.0900e-04	0.99967
β_1	0.999000	0.000032	3.0855e+04	0.00000

From the results in Table 5 it is observed that the mean value is (-0.043001), this indicates a slight downward trend. Moreover, the constant variance term is (0.000850), this indicates low-base line volatility. In addition, the shock coefficient is (0.0000); this indicates that past shocks have a negligible impact on current volatility. Finally, the lagged volatility coefficient is (0.9990), this indicates strong persistence and the ARIMA model removed the autocorrelations.

Model Adequacy Checking

Table 6 ARCH-LM Test Results

Lag	Statistic	Shape	Scale	p.value
Lag 3	0.06784	0.500	2.000	0.7945
Lag 5	1.36927	1.440	1.667	0.6273
Lag 7	1.69024	2.315	1.543	0.7825

From the results in Table 6 it is observed that all the probability values are (0.7945, 0.6273, 0.7825) are greater than the alpha value (0.05). Thus, the residuals of the model are approximately normally distributed. There is no remaining ARCH-effect in the model.

Table 7 Ljun-Box Test Results

Lag	Statistic	p-value = 0.4191
Lag 1	3.331	0.06797
Lag 2	3.384	0.11048
Lag 4	4.351	0.21336

From the results in Table 7 it is observed that all the probability values are (0.06797, 0.11048, 0.21336) are greater than the alpha value (0.05). Thus, there is no significant serial correlation in the residuals, indicating the model reduced volatility persistence.

Forecast Results using ARIMA (1, 1, 1,)-GARCH (1, 1)
Table 8 Three Years a head Forecast Results

Year	Sigma
2023	0.9476
2024	0.9476
2025	0.9475

From the results in Table 8 it is observed that the conditional standard deviation of the forecast error are close (0.9476, 0.9476, 0.9475), this indicates stable and very little change in volatility of currency in circulation. Moreover, this means the risks or fluctuations in currency are predicted to remain low and steady.
From the time series visualization achieved it was found that the time series exhibits trend with structural breaks. The test of stationarity confirmed that the time series has unit root and it was differenced to make it stationary. ARIMA (1, 1) and ARIMA (1, 1 2) were fitted in the study. ARIMA (1, 1, 1) was found to be the best model with least AIC and the residuals of the model were extracted. Presence of heteroscedasticity was checked in the residuals of the ARIMA (1, 1, 1) model and the heteroscedasticity was confirmed. ARIMA (1, 1, 1)-GARCH (1, 1), ARIMA (1, 1, 1)-FIGARCH (1, 1), ARIMA (1, 1, 1)-EGARCH (1, 1), and ARIMA (1, 1, 1)-TGARCH (1, 1) models were fitted. ARIMA (1, 1, 1)-GARCH (1, 1) was found to be best model when compared with least MAE, MSE, and RMSE when compare with the hybrid models. The model was checked and found that no remaining arch effect and serial correlation in the model. From the forecast results obtained it was observed that the conditional standard deviation of the forecast error are identical, this indicates stable volatility of currency in circulation. moreover, the identical forecasts imply that the model does not anticipate significant changes in volatility during these periods. Currency in circulation in Nigeria continues to rise steadily, reaching new hights as documented by Atoyebi (2023). Interestingly, despite this increase, the conditional standard deviation of forecast errors remains stable, suggesting that the volatility of currency is in use, its flucuations are predictable and management, which supports effective monetary policy and liqudity planning.

The forecasted conditional standard deviations, being identical, imply that while volatility is present, it's not excessively high-allowing for smoother monetary policy transmission and reduced risks of financial instability. Central banks should monitor to avoid sudden spikes, as extreme volatility can undermine investors confidence and disrupt economic stabiliy, especially in emerging markets. consitent communication and flexible exchange rate policies can help manage these risks effectively.

CONCLUSION

The time series data consists of trend and with structural breaks over the long period of time. In addition, the time series suffers with fluctuation of mean and variance, these characteristics made the time series not stationary. Moreover, there is presence of heteroscedasticity in the residuals of the time series. Four hybrid time series models were fitted and ARIMA (1, 1, 1)-GARCH (1, 1) was found to be the best model with least AIC, MAE, MSE, and RMSE when compared with the other hybrid models. Furthermore, all the tests carried out to diagnose the residuals of the model suggest that the errors are normally distributed. The currency in circulation has short memory, indicating the time series is not strongly affected by autocorrelation. The lagged volatility coefficient is (0.9990); this indicates strong persistence and the ARIMA model removed the autocorrelations. There is no significant serial correlation in the residuals, indicating the model reduced volatility persistence. Finally the results of the volatility forecast were made. From the forecast results obtained it was observed that the conditional standard deviation of the forecast error are identical with little change, this indicates stable volatility of currency in circulation. moreover, the identical forecasts imply that the model does not anticipate significant changes in volatility during these periods.

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