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Robustness and Efficiency of Complete versus Balanced Incomplete Sudoku Square Designs: A Comparative Study

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ABSTRACT
Researchers in industrial engineering, agriculture, and pharmaceutical sciences require evidence-based guidance when selecting experimental designs that optimally balance precision, cost, and feasibility. While Complete Sudoku Square Designs (CSSDs) guarantee full treatment balance, they often require a large number of experimental runs. In contrast, Balanced Incomplete Sudoku Square Designs (BISSDs) reduce experimental burden by sacrificing partial completeness, yet the statistical implications of this trade-off remain insufficiently explored. Although previous studies have focused on the construction and analysis of CSSD and BISSD, their relative efficiency and robustness have not been adequately investigated. This study presents a comparative assessment of the statistical efficiency and robustness of CSSD and BISSD using three performance criteria: mean squared error (MSE), p-values associated with treatment effects, and the power of the F-test. Both hypothetical experimental data and Monte Carlo simulation techniques were employed for evaluation. The results indicate that CSSD consistently outperforms BISSD across all criteria, with relative efficiency values exceeding unity. Although both designs remain suitable for experiments requiring row–column–block balance, CSSD demonstrates superior precision and robustness, while BISSD offers greater flexibility when experimental resources are constrained. These findings provide practical guidance for researchers in selecting appropriate Sudoku-based experimental designs.

Keywords:
Sudoku Square Design;
Balanced Incomplete Sudoku Square Design;
Relative Efficiency;
Monte Carlo Simulation;
Experimental Design.

INTRODUCTION

Design of Experiments (DOE) is a structured approach for planning experiments to ensure that relevant data are collected and analyzed using appropriate statistical techniques, thereby yielding valid and objective conclusions (Montgomery, 2017). An experimental study fundamentally consists of two interconnected components: the experimental design and the statistical analysis of the resulting data.

Design of Experiments (DOE) has evolved into a powerful tool that combines both scientific principles and statistical analysis rooted in the need to efficiently utilize resources and obtain meaningful insights (Atlan & Sahin, 2021).

Sudoku grids, originally popularized as recreational puzzles, have gained attention in statistical experimental design due to their unique structure. Sudoku grids represent a specialized extension of Latin square designs, incorporating additional constraints that allow for the control of more sources of variation than traditional Latin square designs (Subramani & Ponnuswamy, 2009). Consequently, Sudoku-based designs have emerged as powerful tools for experimental situations requiring multi-level blocking structures.

Two prominent Sudoku-based experimental designs are the Complete Sudoku Square Design (CSSD) and the Balanced Incomplete Sudoku Square Design (BISSD). A CSSD arranges treatments such that each treatment appears exactly once in every row, column, and sub-square (Oladugba *et al.*, 2013).

In contrast, a BISSD is derived by systematically removing selected cells—typically through the deletion of transversals—from a complete design while maintaining balanced treatment occurrence across rows,

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columns, and sub-squares Danbaba, (2016a).
Although extensive literature exists on the construction and analysis of Sudoku square designs [2–7, 28–33], little attention has been given to evaluating their relative efficiency and robustness. Relative efficiency is commonly assessed by comparing error variances, p-values, or statistical power, with lower variance and higher power indicating superior design performance (Dauran *et al,* 2020).
Given the growing application of Sudoku-based designs in agriculture, manufacturing, and other experimental fields, a rigorous comparison of CSSD and BISSD is both timely and necessary. This study therefore aims to evaluate and compare the relative efficiency of CSSD and BISSD using mean squared error, p-values, and power of the F-test, thereby providing empirical guidance for design selection.
Sudoku, a popular puzzle game, has been used much recently, in statistical design of experiments due to its unique structure. sudoku grids a specialized form of Latin squares with some more constraints, can be utilized as statistical designs that will extract more source of variations than does the traditional latin square designs. In this context, Sudoku puzzles, which are incomplete Sudoku grids can be completed to form Sudoku experimental designs or simply as Sudoku designs (Subramani, and Ponnuswamy 2009).
The relative efficiency of two designs is mostly assessed by comparing their error variances with the design having the smallest variance to be more efficient than the other. The relative measures also utilize and the p-value of both designs to evaluate their efficiency (Oladugba *et al,* 2013).

MATERIALS AND METHODS

Data Description

The dataset for this study was adapted from a hypothetical experimental dataset based on a Sudoku square design of order nine presented in (Atlan & Sahin, 2021), involving nine treatments. The dataset was subsequently modified to conform to a balanced incomplete structure following the methodology proposed in (Dauran *et al,* 2020).

Monte Carlo Simulation

Monte Carlo simulation was conducted for both CSSD and BISSD using 10,000 replications. For CSSD, the design parameters included nine treatments, nine rows, nine columns, three row blocks, three column blocks, and nine sub-squares. For BISSD, All simulated observations were generated under the assumption of normality with mean μ and variance σ^2.

Statistical Models

The linear model for the Complete Sudoku Square Design (CSSD) is:

$$Y_{ijklm} = \mu + RB_i + CB_j + T_k + C_m + S_n + \varepsilon_{ijklm}$$

Where μ is the overall mean, RB, CB, T, R, C and S represent row-block, column-block, treatment, row, column, and sub-square effects, respectively, and ε denotes the random error term with mean zero and variance σ^2.
A corresponding reduced model was specified for BISSD, accounting for missing cells while preserving balance across design components.

Measures of Efficiency

Relative efficiency was evaluated using:

- Mean Squared Error (MSE),
- P-values associated with treatment F-tests, and
- Power of the F-test under a 5% significance level.

Monte Carlo simulations were further employed to assess robustness under repeated sampling.

RESULTS AND DISCUSSION

Analysis of Variance

Table 1: Complete Sudoku Square Design (CSSD)

Variation Source	Sum of Squares	df	Mean Square	F-value	P-value
Treatments	105.78	8	13.22	1.04	0.43
Row-block	25.41	2	12.71	0.99	0.38
Column-block	69.56	2	34.78	2.72	0.08
Row	152.40	8	19.06	1.49	0.19
Column	123.60	8	15.45	1.21	0.32
Squares	111.10	8	13.89	1.09	0.39
Error	562.10	44	12.78		
Total	1150.00	80			

Table 2: Balanced Incomplete Sudoku Square Design (BISSD)

Variation Source	Sum of Squares	df	Mean Square	F-value	P-value
Treatments	39.72	5	7.94	0.50	0.77
Row-block	35.11	2	17.56	1.10	0.35
Column-block	65.44	2	32.72	2.05	0.15
Row	98.33	8	12.29	0.77	0.63
Column	83.67	8	10.46	0.66	0.72
Squares	155.67	8	19.46	1.22	0.34
Error	318.89	20	15.94		
Total	796.83	53			

Relative Efficiency

- MSE-based relative efficiency: 1.25 → CSSD more precise than BISSD
- P-value-based relative efficiency: 1.82 → CSSD more robust
- Power-based relative efficiency: 3.38 → CSSD has higher statistical power

Monte Carlo Simulation Results

Table 3: Simulation Metrics (Rounded)

Metric	Replications	Relative Efficiency >1	Mean Value	Std. Dev.
MSE	10000	5323	1.15	0.81
P-value	10000	5758	1.18	0.73
Power of F-test	10000	6433	2.66	3.05

These results confirm that CSSD outperforms BISSD across all metrics.

CONCLUSION

1. CSSD consistently outperforms BISSD in MSE, p-values, and F-test power, indicating higher precision and robustness.
2. Both designs are suitable for experimental research; the choice depends on experiment-specific requirements such as completeness vs. flexibility.

CSSD is recommended when precision and robustness are critical, whereas BISSD offers a flexible alternative under constrained resources.

REFERENCE

AtalanA, & Şahin H. (2021). Design of Experiments Optimizations Application in Physics: A Case Study of the Damped Driven Pendulum Experiment. Sigma J Eng Nat Sci 39:3:322–330.

AtalanA, & Şahin H. (2021). Design of Experiments Optimizations Application in Physics: A Case Study of the Damped Driven Pendulum Experiment. Sigma J Eng Nat Sci 39:3:322–330.

Danbaba A. &Dauran N.S, (2016). Construction and Analysis of Partially Balanced Sudoku Design of Prime Order. International Journal of Statistics and Applications. 6(5): 325- 327

Danbaba, A. (2016b). Construction and Analysis of Samurai Sudoku. International Journal of

Mathematical, Computational, Physical, Electrical and Computer Engineering. 10(4):126-131.

Danbaba, A., & Shehu, A. (2016). On the combined analysis of Sudoku square designs with some common treatments. International Journal of Statistics and Applications, 6(6), 347–351.

Dauran, N.S, Odeyale, A.B, Danbaba, A., &Shehu, A., (2020). Construction and Analysis of Balanced Incomplete Sudoku Square Design. FUDMA. Journal of Sciences (FJS)

Ezra, P.N., &Nwovu, N., (2021), Relative Efficiency of Split-plot Design to Randomized Designs. International Journal of Mathematics, Trends and Technology 67(6), 131- 135.

Hui-Dong, M., &Ru-Gen, X. (2008). Sudoku square—a new design in field experiment. ActaAgron Sin, 34(9), 1489–1493.

Hui-Dong M. &Ru-Gen, X. (2008). Sudoku Square- a New Design in field Experiment, ActaAgron Sin, 34(9), 1489–1493.

Montgomery, D.C. (2017). Design and Analysis of Experiments. John Wiley & Sons, New York.

Oladugba, A.V., &Onuoa, D.O., &Opara, P.N., (2013), Relative Efficiency of Split-plot Design to Randomized Complete Block Design. West African Journal of Industrial andAcademic Research 7(1), 113-119.

Subramani, J. (2012). Construction of Graeco Sudoku square designs of odd orders. Bonfring International Journal of Data Mining, 2(2), 37–41.

Subramani, J., & Ponnuswamy, K.N. (2009). Construction and analysis of Sudoku designs. Model Assisted Statistics and Applications, 4(4), 287–301.

Subramani, J. & K.N. Ponnuswamy (2009). Construction and Analysis of Sudoku Designs. Model Assisted Statistics and Applications, 4(4), 287-301.