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Using the Generalized Sundman Transformation to Achieve Linearization of the General Modified Second-Order Lane-Emden Differential Equation

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ABSTRACT
The generalized Sundman transformation is a mathematical technique designed to simplify the integration of differential equations, particularly in fields like dynamical systems and celestial mechanics. This powerful method helps transform complicated dynamical equations into forms that are easier to analyze or solve numerically, especially when dealing with challenging singularities. Among the various nonlinear second-order differential equations, the general modified Emden equation (GMEE) is notable for its frequent appearance across multiple areas of applied mathematics and physics. This equation is a variation of the classic Emden-Fowler equation, which is commonly used to model thermodynamics, stellar structure, and other physical phenomena. Its nonlinear nature allows it to effectively represent the complexities found in real-world systems across diverse fields, making it highly versatile. This study examines the generalized modified Lane-Emden equation derived from the general Lane-Emden differential equation. Using the generalized Sundman transformation approach, exact solutions are obtained for the second-order general modified Lane-Emden differential equation through analytical linearization. Additionally taken into consideration were a few particular instances of the modified Lane-Emden differential equations and their solutions.

INTRODUCTION

One of the most intricate and thoroughly studied nonlinear dynamic equations in the literature is the Emden differential equation. It has applications across a range of fields including celestial mechanics, fluid dynamics, stellar structure, isothermal gas spheres, thermionic currents, and so on (Orverem et al., 2021). In a separate study, approximate analytical solutions for nonlinear Emden-Fowler type equations were derived using the differential transform method (DTM) (Kartak, 2011). The DTM serves as both a numerical and analytical technique for solving integral equations, as well as ordinary and partial differential equations.

In another study, the authors aimed to find solutions for the Lane-Emden equation, a well-known and challenging nonlinear dynamic equation given by $y''(x) + \frac{2}{x}y'(x) + y^n = 0$for $n = 0, 1, 2, 3, 4$ and 5, using the relatively new exact series technique called the differential transform method (DTM). The Lane-Emden equation models various phenomena in theoretical physics and astrophysics (Mukherjee et al., 2011).

Additionally, singular initial value problems related to a new class of Lane-Emden or Emden-Fowler type equations were explored in (Biles et al., 2008).

A specific second-order Lane-Emden differential equation was solved using various methods, including He's variational iteration method, the adomian decomposition method, the homotopy analysis method, the homotopy perturbation method, and the finite difference method (Yuksel & Gozukizil, 2023). The solutions obtained from these methods were compared to evaluate which one provides the most accurate and practical results.

The modified Emden equation plays a significant role in analyzing heat distribution within spherical objects, which is especially relevant in fields like plasma physics and astrophysics. It models how heat is conducted and how temperature varies within such bodies. In thermodynamic systems exhibiting spherical symmetry particularly those involving irreversible processes that generate entropy, the equation also helps describe entropy distribution.

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Additionally, certain cosmological models that address the universe's evolution incorporate this equation. Under particular assumptions about how matter and energy are distributed, it contributes to understanding the dynamics of the universe's expansion.

Motsa & Shateyi, (2012) suggested a novel way to solve singular initial and boundary value problems of the Lane-Emden type using the successive linearization method. The results of previous approaches in the literature and precise analytical answers were compared in order to show the dependability of the suggested approach. It was discovered that the approach works better than some numerical techniques, is simple to use, and produces accurate results.

Another article examines how to solve singular Initial Value Problems (IVPs) of the Lane-Emden type in second-order Ordinary Differential Equations using the Differential Transformation Method (DTM) to get both exact and approximate solutions (Merdan & Yildirim, 2011). The method is straightforward to apply to a wide range of linear and nonlinear problems, significantly reducing computational effort while yielding series solutions with a rapid convergence rate. In some cases, exact solutions can be derived directly from the series form. The findings demonstrate that DTM is an efficient, reliable, user-friendly, and accurate approach.

The general modified Emden equation is frequently used in diffusion models for porous media, especially when diffusion is driven by variations in temperature, pressure, or concentration. It effectively describes diffusion processes within spherical environments, such as in gas adsorption scenarios. In catalytic reactors that contain spherical catalysts, the equation is instrumental in analyzing how the reaction rate depends on both the concentration of reactants and their diffusion into the catalyst material. Moreover, the equation is applied in the investigation of gravitational collapse, where massive celestial bodies like stars collapse under the force of their own gravity. It helps explain the distribution of mass and pressure during such events, offering valuable understanding of the formation of neutron stars and black holes.

Using the Mittag-Leffler kernel and the Atangana-Baleanu-Caputo (ABC) fractional derivative, this work examines the dynamics of HIV/AIDS transmission. To prove the existence and uniqueness of the model's solution, the Picard-Lindelöf approach was used (Ezugorie & Micheal, 2024).

Orhan et al., (2020) demonstrated that arbitrary coefficients α and β have invariant solutions in the modified Emden equation. First, they showed that it is possible to linearize the modified Emden equation. Once this equation is linearized, a workable approach can be used to determine the symmetries of the equation. With the use of these symmetries and a new algorithm, the exact solutions to the problem were obtained. Furthermore, determining solutions was categorized according to the arbitrary coefficients' physical meaning. Lastly, all of the solution visualizations were displayed using Matlab and Mathematica.

A study's objective was to present a new model based on the nonlinear singular second order delay differential equation of the Lane–Emden type that was successfully solved numerically using the heuristic technique by Sabir et al., (2021). This paper presented four different examples, namely genetic algorithms (GA), sequential quadratic programming (SQP), and GA-SQP, which were numerically resolved using artificial neural networks optimized by the global search, local search, and their hybrid combinations, respectively, and based on the designed model. The performance and accuracy of the suggested heuristic technique were demonstrated by comparing the numerical results of the constructed model with the exact/explicit outcomes. Additionally, statistical analyses and evaluations were provided about the precision and effectiveness of the model that was created using heuristic methods.

The equation can be used to forecast population dynamics in mathematical biology when nonlinear growth and interaction factors are present. It describes how populations fluctuate over time while taking into consideration growth rates that are impacted by both population density and outside factors. The general modified Emden equation can be used to characterize the behavior of some nonlinear oscillatory systems. Both mechanical and electrical engineering can benefit from systems that exhibit oscillations with amplitudes and frequencies that change over time. It can be applied to the study of chaotic systems, in which nonlinear interactions cause complex, unpredictable behaviors over time.

The boundary value approach was used by Okunuga et al., (2012) to resolve Lane-Emden type second order non-linear ordinary differential equations. The multistep collocation technique was used to derive a class of second derivative backward differentiation formulas from certain continuous multistep schemes. The method transforms the numerical methods into a set of non-linear equations represented as a tridiagonal matrix, allowing numerical solutions to be found simultaneously on the whole range of integration. Both the stability properties and general properties of the numerical approach were shown. To illustrate the method's effectiveness, a few Lane-Emden type equations were solved.

Previously, Duarte, Moreira, and Santos used the Laguerre form to examine the linearization problem of a second-order ordinary differential equation via the generalized Sundman transformation (Nakpim & Meleshko, 2010). The authors demonstrated that the Laguerre form of linearization $u'' = 0$ is not sufficient. From the perspective of the generalized Sundman transformation, the linearization of a class of nonlinear second-order ordinary differential equations of Liénard type was considered. The generalized Sundman transformation that linearizes the class of equations was built (Johnpillai & Mahomed, 2013). A novel description of S-linearizable equations using one auxiliary function and the ODE coefficients. By explicitly obtaining the general solutions for the first integral using this new criterion, a direct alternative method for building the first integrals and Sundman transformations is provided (Mustafa et al., 2013).

This study is the first to use the generalized Sundman transformation (GST) to linearize the general modified Emden differential equation of second order. The many uses of the general modified Emden differential equation serve as the driving force behind this study. This work uses the generalized Sundman transformation strategy to linearize the second-order generalized modified Emden differential equation.

MATERIALS AND METHODS

The Generalized Sundman Transformation (GST)

A nonpoint transformation given as

$$u(t) = F(x,y),\ dt = G(x,y)dx,\ F_yG \neq 0, \quad (1)$$

is known as the generalized Sundman transformation.

The necessary format for an ordinary differential equation of second order $y'' = \mu(x,y,y')$, that can be linearized to become a linear ordinary differential equation

$$u'' + \beta u' + \alpha u = \gamma, \quad (2)$$

by means of the transformation (1), is provided by

$$y'' + \mu_2 {y'}^2 + \mu_1 y' + \mu_0 = 0, \quad (3)$$

where in equation (2), $\alpha(t), \beta(t)$ and $\gamma(t)$ represent various functions.

Consider the case $\mu_3 \neq 0$ and $\mu_5 \neq 0$, where $\mu_3 = \mu_{1y} - 2\mu_{2x}$, $\mu_4 = 2\mu_{0yy} - 2\mu_{1xy} + 2\mu_0\mu_{2y} - \mu_{1y}\mu_1 + 2\mu_{0y}\mu_2 + 2\mu_{2xx}$ and $\mu_5 = \mu_{2xx} + \mu_{2x}\mu_1 + \mu_{3x} + \mu_1\mu_3$.

The following prerequisites must be met in order for equation (3) to be linearizable by (1):

$$\mu_{0x}\mu_3 = 2\mu_0(\mu_5 - \mu_1\mu_3), \quad (4)$$

$$\mu_{2xxy}\mu_3 = -\mu_{2xy}\mu_1\mu_3 - \mu_{3xy}\mu_3 - 2\mu_{2x}^2\mu_3 - 2\mu_{2x}\mu_3^2 - \mu_{3y}\mu_1\mu_3 + \mu_{3y}\mu_5, \quad (5)$$

$$\mu_{2xxx}\mu_3 = -\mu_{3xx}\mu_3 - \mu_{1x}\mu_{2x}\mu_3 - \mu_{1x}\mu_3^2 + \mu_{2x}\mu_1^2\mu_3 + \mu_1^2\mu_3^2 - 2\mu_1\mu_3\mu_5 + \mu_5(\mu_{3x} + \mu_5), \quad (6)$$

and

$$\mu_3\mu_5\big(6\mu_{0y}\mu_{2x} + 2\mu_{2xy}\mu_0 + 4\mu_{2x}\mu_0\mu_2 + 2\mu_{3y}\mu_0 + 4\mu_0\mu_2\mu_3 + \mu_1\mu_5\big) - \mu_3^2\big(6\mu_{2x}^2\mu_0 + 12\mu_{2x}\mu_1\mu_3 - 6\mu_{0y}\mu_5 + 6\mu_0\mu_3^2\big) - \mu_4\mu_5^2 - 2\mu_5^3 = 0. \quad (7)$$

The following equations must be solved in order to obtain the F and G functions:

$$F_x = 0, \quad (8)$$

$$F_{yy} = \frac{F_yG_y + \mu_2F_yG}{G}, \quad (9)$$

$$G_x = \frac{G(\mu_{2xx} + \mu_{2x}\mu_1 + \mu_{3x})}{\mu_3}, \quad (10)$$

$$G_y = \frac{G\mu_3(\mu_{2x} + \mu_3)}{\mu_5}. \quad (11)$$

The following equations can be used to find the constants α, β, and γ from equation (2):

$$\alpha = \frac{G(\mu_{0y} + \mu_0\mu_2) - G_y\mu_0}{G^3}, \quad (12)$$

$$\beta = \frac{G_x + G\mu_1}{G^2}, \quad (13)$$

$$\gamma = \frac{\alpha FG^2 - F_y\mu_0}{G^2}. \quad (14)$$

RESULTS AND DISCUSSION

According to Berkovic (1997), the generic second-order Lane-Emden equation is

$$y'' + a_1(x)y' + a_0(x)y + f(x)y^n = 0, n \neq 0, n \neq 1. \quad (15)$$

Assuming that the coefficients in (15) are

$$a_1(x) = \alpha y, a_0(x) = 0, f(x) = \beta, n = 3, \quad (16)$$

we have that

$$y'' + \alpha y y' + \beta y^3 = 0, \quad (17)$$

as the general modified Lane- Emden differential equation.

From equation (17), the coefficients of the general modified Lane-Emden equation are:

$$\mu_0 = \beta y^3, \qquad \mu_1 = \alpha y, \qquad \mu_2 = 0$$

and

$$\mu_3 = \mu_{1y} - 2\mu_{2x} = \alpha \neq 0,$$

$$\mu_4 = 2\mu_{0yy} - 2\mu_{1xy} + 2\mu_0\mu_{2y} - \mu_{1y}\mu_1 + 2\mu_{0y}\mu_2 + 2\mu_{2xx} = 12\beta y - \alpha^2 y,$$

$$\mu_5 = \mu_{2xx} + \mu_{2x}\mu_1 + \mu_{3x} + \mu_1\mu_3 = \alpha^2 y \neq 0.$$

One now checks to see if $\mu_0, \mu_1, \mu_2, \mu_3$, μ_4 and μ_5 satisfy equations (4) to (7). From equation (4) we see that $\mu_{0x}\mu_3 = 0$ and

$$2\mu_0(\mu_5 - \mu_1\mu_3) = 2\beta y^3(\alpha^2 y - \alpha^2 y) = 0.$$

That is, equation (4) is satisfied. From equation (5),

$$\mu_{2xxy}\mu_3 = -\mu_{2xy}\mu_1\mu_3 - \mu_{3xy}\mu_3 - 2\mu_{2x}^2\mu_3 - 2\mu_{2x}\mu_3^2 - \mu_{3y}\mu_1\mu_3 + \mu_{3y}\mu_5 = 0,$$

and from (6), $\mu_{2xxx}\mu_3 = 0$ and the right-hand side

$$-\mu_{3xx}\mu_3 - \mu_{1x}\mu_{2x}\mu_3 - \mu_{1x}\mu_3^2 + \mu_{2x}\mu_1^2\mu_3 + \mu_1^2\mu_3^2 - 2\mu_1\mu_3\mu_5 + \mu_5(\mu_{3x} + \mu_5) = \alpha^4 y^2 - 2\alpha^4 y^2 + \alpha^4 y^2 = 0.$$

Thus, equation (6) is also satisfied.

From equation (7), one has that

$$\mu_3\mu_5(\mu_1\mu_5) - \mu_3^2\big(-6\mu_{0y}\mu_5 + 6\mu_0\mu_3^2\big) - \mu_4\mu_5^2 - 2\mu_5^3 = 0.$$

This becomes

$$\alpha^6 y^3 - \alpha^2(-12\alpha^2\beta y^3) - 12\alpha^4\beta y^3 + \alpha^6 y^3 - 2\alpha^6 y^3 = 0.$$

Since all the four equations above are satisfied, the general modified Emden equation (17) through the generalized Sundman transformation (GST), can be linearized.

Next, one finds expressions for F and G that will satisfy equations (8)-(11), and these expressions are:

$$F = y^2, \; G = y. \tag{18}$$

One now proceeds to find α, β and γ as given in equations (12), (13) and (14). From these equations, one has: $\alpha = 2\beta, \; \beta = \alpha$ and $\gamma = 0$. With these, the linear equation (2) becomes

$$u'' + \alpha u' + 2\beta u = 0. \tag{19}$$

The characteristics equation of equation (19) is

$$r^2 + \alpha r + 2\beta = 0. \tag{20}$$

Solving equation (20) with the aid of quadratic formula, we have

$$r = \frac{-\alpha \pm \sqrt{\alpha^2 - 8\beta}}{2}, \tag{21}$$

where $r_1 = \frac{-\alpha + \sqrt{\alpha^2 - 8\beta}}{2}, \; r_2 = \frac{-\alpha - \sqrt{\alpha^2 - 8\beta}}{2}$.

Now, one has three cases depending on the discriminant $D = \alpha^2 - 8\beta$:

Case 1: If $D = \alpha^2 - 8\beta > 0$, the roots r_1 and r_2 are real and distinct. In this case, equation (19) has the following solution:

$$u(t) = c_1 e^{\frac{-\alpha + \sqrt{\alpha^2 - 8\beta}}{2} t} + c_2 e^{\frac{-\alpha - \sqrt{\alpha^2 - 8\beta}}{2} t},$$

where the constants c_1 and c_2 are arbitrary.

Once the generalized Sundman transformation $u(t) = F(x, y), \; dt = G(x, y)dx$, (from (18)) is applied,

$$y^2 = u(t), \qquad dt = y dx$$

is obtained.

One now has:

$$y^2 = c_1 e^{\frac{-\alpha + \sqrt{\alpha^2 - 8\beta}}{2} t} + c_2 e^{\frac{-\alpha - \sqrt{\alpha^2 - 8\beta}}{2} t},$$

so that

$$y = \sqrt{c_1 e^{\frac{-\alpha + \sqrt{\alpha^2 - 8\beta}}{2} \phi(x)} + c_2 e^{\frac{-\alpha - \sqrt{\alpha^2 - 8\beta}}{2} \phi(x)}},$$

where $\phi(x)$ is the solution of the equation $dt = y dx$.

Case 2: If $D = \alpha^2 - 8\beta = 0$, then the roots are real and equal, $r_1 = r_2 = \frac{-\alpha}{2}$. In this case, the general solution to equation (19) is:

$$u(t) = (c_1 + c_2 t) e^{\frac{-\alpha}{2} t},$$

where c_1 and c_2 are arbitrary constants.

From case 2, on application of the GST established in equation (18), one has that

$$y = \sqrt{(c_1 + c_2 t) e^{\frac{-\alpha}{2} \phi(x)}},$$

where $\phi(x)$ is the solution of the equation $dt = y dx$.

Case 3: If $D = \alpha^2 - 8\beta < 0$, then the roots are complex, of the form $r = \alpha \pm i\beta$, where: $\alpha = \frac{-\alpha}{2}, \; \beta = \frac{\sqrt{8\beta - \alpha^2}}{2}$. In this case, the general solution of (19) is

$$u(t) = e^{\frac{-\alpha}{2} t} \left(c_1 \cos\left(\frac{\sqrt{8\beta - \alpha^2}}{2} t \right) + c_2 \sin\left(\frac{\sqrt{8\beta - \alpha^2}}{2} t \right) \right).$$

Lastly, case 3 has the solution on application of the GST in (18) as:

$$y = \sqrt{e^{\frac{-\alpha}{2} \phi(x)} \left(c_1 \cos\left(\frac{\sqrt{8\beta - \alpha^2}}{2} \phi(x) \right) + c_2 \sin\left(\frac{\sqrt{8\beta - \alpha^2}}{2} \phi(x) \right) \right)},$$

where $\phi(x)$ is the solution of the equation $dt = y dx$.

From the general modified Lane-Emden equation established in equation (17), if $\alpha = \beta = 1$, one has that:

$$y'' + y y' + y^3 = 0, \tag{22}$$

with the coefficients: $\mu_0 = y^3, \; \mu_1 = y, \; \mu_2 = 0$, and one finds that $\mu_3 = 1 \neq 0, \; \mu_4 = 11y$ and $\mu_5 = y \neq 0$.

Next, one tests to see if these coefficients satisfy the linearizability conditions (4) to (7). One can see that all the linearizability conditions are satisfied, therefore, equation (22) is linearizable using this method of the generalized Sundman transformation (GST). Taking

$$F = y^2, \qquad G = y,$$

one sees that equations (8) through (11) are fully satisfied. To obtain α, β and γ from equations (12) to (14), one has that $\alpha = 2, \; \beta = 1$ and $\gamma = 0$ respectively. The linear equation in (2) becomes

$$u'' + u' + 2u = 0, \tag{23}$$

with the solution

$$u(t) = e^{-\frac{t}{2}} \left(c_1 \cos \frac{\sqrt{7}}{2} t + c_2 \sin \frac{\sqrt{7}}{2} t \right).$$

Applying the GST, one obtains that

$$y = \left[e^{-\frac{t}{2}} \left(c_1 \cos \frac{\sqrt{7}}{2} t + c_2 \sin \frac{\sqrt{7}}{2} t \right) \right]^{\frac{1}{2}},$$

and finally

$$y = \left[e^{-\frac{\phi(x)}{2}} \left(c_1 \cos \frac{\sqrt{7}}{2} \phi(x) + c_2 \sin \frac{\sqrt{7}}{2} \phi(x) \right) \right]^{\frac{1}{2}},$$

where $\phi(x)$ is the solution of the equation $dt = y dx$.

In another instance, Orverem et al., (2021) considered the modified Emden equation of the form

$$y'' + a y y' + \frac{a^2}{9} y^3 = 0, \tag{24}$$

where $\alpha = a$, and $\beta = \frac{a^2}{9}$.

In this case, the linearized equation is

$$u'' + a u' + \frac{2a^2}{9} u = 0, \tag{25}$$

with the general solution

$$u = c_1 e^{-\frac{at}{3}} + c_2 e^{-\frac{2at}{3}}. \tag{26}$$

Applying the generalized Sundman transformation produced the following outcome

$$y(x) = \sqrt{c_1 e^{-\frac{a}{3} \phi(x)} + c_2 e^{-\frac{2a}{3} \phi(x)}}. \tag{27}$$

A solution to the equation

$$\frac{dt}{dx} = \sqrt{c_1 e^{-\frac{a}{3} t} + c_2 e^{-\frac{2a}{3} t}}$$

is the function $t = \phi(x)$.

CONCLUSION

This work uses the generalized Sundman transformation strategy to linearize the general modified Lane-Emden nonlinear second order ordinary differential equation. The conventional second order linear homogeneous differential equations with constant coefficients, where the characteristic equations that emerged from the linearization process were solved. The general solution of the general modified Emden differential equation can be found by applying the generalized Sundman transformation. Some specific cases of the modified Lane-Emden differential equations and their solutions were also considered.

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