Journal of Basics and Applied Sciences Research (JOBASR)
ISSN (print): 3026-9091, ISSN (online): 1597-9962
Volume 4(1) January 2026
DOI: https://dx.doi.org/10.4314/jobasr.v4i1.23

A Lotka-Volterra Approach to Modelling Revenue and Consumer Dynamics in Electricity Distribution: A Case Study of KEDCO

Lawal, Nafiu[1], Baoku, Ismail Gboyega[2*] & Yusuf, Auwal Bichi[3]
[1,2,3] Department of Mathematics, Federal University Dutsin-Ma, Katsina State
[*]Corresponding Author Email: ibaoku@fudutsinma.edu.ng

ABSTRACT

This study presents a two-equation predator-prey model to capture the nonlinear interactions between consumer growth and revenue dynamics in electricity distribution systems. Drawing on the classical Lotka-Volterra framework, total active consumers are modelled as the "prey" population sustaining revenue, while revenue generation functions as the "predator," exerting feedback effects through pricing signals and service delivery. The model is calibrated using Kano Electricity Distribution Company (KEDCO)'s operational data, formatted as quarterly time-series data from 2015 to 2023. Parameters were initially estimated using Nonlinear Least Squares Regression (NLSR), and the system was numerically solved using the Runge-Kutta 4th Order (RK4) method. However, the NLSR approach produced unstable forecasts with economically unrealistic equilibria, highlighting its limitations for complex, nonlinear, and large-scale systems. As a result, parameters were subsequently refined using Differential Evolution with logarithmic transformation to ensure numerical stability and economic plausibility. In contrast, these refined parameters generated stable and accurate forecasts, achieving mean absolute percentage errors of 0.81% for consumers and 10.34% for revenue. Equilibrium and Sensitivity analysis were conducted which confirmed neutrally stable centres characteristic of Lotka-Volterra systems, but crucially, only the refined model yielded economically plausible equilibria. The Sensitivity analysis further highlighted the model's responsiveness to operational inefficiencies and pricing policies, revealing that consumer growth is most influenced by intrinsic growth and interaction rates while revenue dynamics depend strongly on decay and consumer contribution rates. This proposed framework demonstrates its utility as a robust, policy-informing tool for optimizing revenue sustainability and demand management in electricity distribution networks.

Keywords:
Lotka-Volterra model,
Electricity distribution,
Nonlinear dynamics,
Parameter estimation,
Revenue forecasting

INTRODUCTION

Electricity distribution is central to socio-economic development, yet distribution companies often face persistent challenges in aligning consumer demand with revenue generation. In Nigeria, the electricity sector is plagued by inefficiencies such as technical losses, energy theft, and estimated billing practices that undermine financial sustainability (Abe et al., 2021; Amadi et al., 2016). Understanding the dynamics between consumer growth and revenue recovery is therefore critical to ensuring reliable service delivery and long-term viability of distribution companies. Nigeria's electricity sector is characterized by an interplay of generation, transmission, and distribution.

After privatization in 2013, ownership of generation and distribution companies transferred to private investors, while the transmission company remained under government control. The sector battles with chronic supply shortages, high aggregate technical and commercial losses, and weak cost recovery mechanisms (Amadi et al., 2016; Edomah et al., 2021). Distribution companies (DisCos), which are the final interface with consumers struggle with revenue leakages due to estimated billing, low metering penetration, and non-payment culture (Adebayo & Ainah, 2024).

Kano Electricity Distribution Company (KEDCO) serving Kano, Katsina, and Jigawa States, covers one of the largest customer bases in the country (KEDCO, n.d.).

How to cite this article: Lawal N., Baoku I. G. & Yusuf A. B. (2026). A Lotka-Volterra Approach to Modelling Revenue and Consumer Dynamics in Electricity Distribution: A Case Study of KEDCO. *Journal of Basics and Applied Sciences Research*, *4*(1), 211-219. https://dx.doi.org/10.4314/jobasr.v4i1.23

Like other DisCos, KEDCO faces the dual challenge of expanding consumer access while ensuring sufficient revenue recovery to sustain operations. This makes KEDCO a suitable case study for investigating consumer-revenue dynamics. Mathematical models provide valuable tools for analysing such complex interactions. Forecasting electricity demand and revenue has received extensive attention. ARIMA and ARIMAX models have been widely employed to capture consumption trends in Nigeria (Maku et al., 2023). While review studies have emphasized their predictive utility (Efekemo et al., 2022), these models however primarily emphasize temporal correlations and do not explicitly capture structural interdependence between consumers and revenue. Contrary to this, predator-prey modelling provides a framework to represent nonlinear feedback mechanisms. Mitropoulou et al. (2022) used such models to analyse competition dynamics in Greece's electricity market, illustrating how cyclical interactions can be uncovered. The theoretical foundations lie in classical ecology (Lotka, 1925; Volterra, 1926) and mathematical biology (Murray, 2002). Broader contributions in epidemiology and economics (Brauer & Castillo-Chavez, 2011; Hritonenko & Yatsenko, 2010) highlight the generality of this approach.
Our study builds on these traditions by adapting a modified Lotka-Volterra predator-prey model, originally formulated by Lotka (1925) and Volterra (1926) for ecological systems, to capture the dynamic relationship between electricity consumers and revenue in Nigeria's electricity sector, specifically KEDCO. This model captures nonlinear feedback mechanisms and equilibrium behaviours, offering insights beyond conventional time-series approaches. The framework is well-suited for KEDCO's demand-revenue dynamics because it describes cyclical, interdependent processes where growth in one variable both sustains and constrains the other (Murray, 2002).

MATERIALS AND METHODS

The methodology centres on applying the Lotka-Volterra predator-prey framework to model KEDCO's demand-revenue interactions showing the cyclical and interdependent behaviour where consumer growth sustains revenue, while revenue policies impact consumer activity. Our approach models consumer demand as the "prey," while revenue represents the "predator" that depends on consumer activity. Revenue-driven factors such as tariff adjustments, billing enforcement and service quality shaped by financial performance can in turn, influence consumer growth, creating a feedback loop. To estimate parameters, Nonlinear Least Squares Regression (NLSR) was first carried out to obtain baseline estimates, and then parameters were refined through Differential Evolution (Ahmad et al., 2022) with logarithmic transformation (Benoit, 2011) for stability. Besides its interpretability, this modelling approach enables equilibrium, stability, and sensitivity analyses that provide insights into system behaviour under different operational or policy scenarios. Quarterly data covering Q1 to Q34 (2015-2023), consisting of total consumer numbers and corresponding revenue figures were obtained from KEDCO operational records published by the National Bureau of Statistics (NBS) of Nigeria. The dataset was cleaned and organized into a time-series format appropriate for dynamic modelling. Observations from Q1-Q30 were used for parameter estimation, while Q31 to Q34 were reserved as an out-of-sample validation set to evaluate the forecasting performance of the model (Chaku et al., 2025).

Model Formulation
The dynamics between consumers and revenue are represented through a modified Lotka-Volterra predator-prey model:

$$\frac{dC}{dt} = rC - \alpha CK \tag{1}$$

$$\frac{dK}{dt} = -\delta K + \beta CK \tag{2}$$

Table 1. Definition of terms

$\boldsymbol{C(t)}$	number of consumers at time $\boldsymbol{t}$
$\boldsymbol{K(t)}$	total revenue at time t
$\boldsymbol{r > 0}$	intrinsic growth rate of consumers (per unit time)
$\boldsymbol{\alpha > 0}$	rate at which revenue generation affects consumer growth
$\boldsymbol{\delta > 0}$	natural decay rate of revenue in absence of consumer activity
$\boldsymbol{\beta > 0}$	rate at which consumers contribute to revenue growth

Equation (1) models consumer dynamics:

- The term rC represents natural growth of the consumer base.
- The term $-\alpha CK$ captures the dampening effect of revenue collection pressure or economic constraints on consumer growth.

Equation (2) models revenue dynamics:

- The term $-\delta K$ accounts for revenue decay due to operational costs, inefficiencies, and non-payment.
- The term $+\beta CK$ represents revenue growth proportional to consumer engagement and transactions.

The model assumes bilinear interaction terms (CK) consistent with classical Lotka-Volterra structure, ensuring that interaction effects vanish if either variable is zero.

Model Assumptions

The modified Lotka-Volterra framework is based on the following key assumptions:

- Consumers grow at an intrinsic rate (r) in the absence of revenue effects.
- Revenue depends entirely on consumer activity and decays naturally at rate (δ) due to inefficiencies and non-payment.
- Revenue pressures (e.g., tariffs, billing) dampen consumer growth.
- Consumers contribute uniformly to revenue growth at rate (β), without distinguishing customer categories whether they are metered or estimated customers.
- Consumer and revenue dynamics are modelled as continuous processes using differential equations, with no explicit carrying capacity constraint such as infrastructural capacity or market saturation.

Equilibrium and Stability Analysis

Equilibrium analysis involved solving for the steady-state conditions of consumers and revenue, while stability was assessed using the Jacobian eigenvalues at the coexistence point.

Setting $\frac{dC}{dt} = 0$ and $\frac{dK}{dt} = 0$ yields equilibrium points:

- Trivial equilibrium: (0,0)
- Non-trivial equilibrium:

From $\frac{dC}{dt} = 0$,

$$rC - \alpha CK = 0 \Rightarrow \mathrm{C} = 0 \text{ or } \mathrm{K} = \frac{r}{\alpha} \quad (3)$$

From $\frac{dK}{dt} = 0$

$$-\delta K + \beta CK = 0 \Rightarrow \mathrm{K} = 0 \text{ or } \mathrm{C} = \frac{\delta}{\beta} \quad (4)$$

The non-trivial coexistence equilibrium

$(C^*, K^*) = \left(\frac{\delta}{\beta}, \frac{r}{\alpha}\right)$

Let $f_1 = rC - \alpha CK$ and $f_2 = -\delta K + \beta CK$

$\frac{\partial f_1}{\partial C} = r - \alpha K$ $\quad$ $\frac{\partial f_1}{\partial K} = -\alpha C$

$\frac{\partial f_2}{\partial C} = \beta K$ $\quad$ $\frac{\partial f_2}{\partial K} = -\delta + \beta C$

The Jacobian matrix at equilibrium is given by

$$J(C,K) = \begin{bmatrix} \partial f_1/\partial C & \partial f_1/\partial K \\ \partial f_2/\partial C & \partial f_2/\partial K \end{bmatrix} \quad (5)$$

$$J(C,K) = \begin{bmatrix} r - \alpha K & -\alpha C \\ \beta K & -\delta + \beta C \end{bmatrix} \quad (6)$$

The Jacobian at coexistence $(C^*, K^*) = \left(\frac{\delta}{\beta}, \frac{r}{\alpha}\right)$ becomes:

$$J(C^*, K^*) = \begin{bmatrix} 0 & -\alpha\frac{\delta}{\beta} \\ \beta\frac{r}{\alpha} & 0 \end{bmatrix} \quad (7)$$

Eigenvalue Analysis

The eigenvalue is computed as $\det(J - \lambda I) = 0$ (8)

But $\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$ (9)

So, $\det(J - \lambda I) = \begin{bmatrix} 0 - \lambda & -\alpha\frac{\delta}{\beta} \\ \beta\frac{r}{\alpha} & 0 - \lambda \end{bmatrix}$ (10)

The characteristic equation is $\lambda^2 + \delta r = 0$ (11)

At coexistence, eigenvalues are purely imaginary

$(\lambda = \pm i\sqrt{\delta r})$ (12)

These purely imaginary eigenvalues imply that the coexistence equilibrium is a **centre**, exhibiting **neutral stability**. This is consistent with classical Lotka-Volterra dynamics, trajectories form closed orbits around the equilibrium with neither exponential growth nor decay. Nonlinearities beyond linearization determine the long-term trajectory behaviour.

Sensitivity Analysis

Local sensitivity analysis was conducted by computing partial derivatives of the model equations with respect to each parameter. This quantified how changes in each parameter influenced consumer and revenue trajectories, helping to identify the most critical drivers of system which most strongly influence consumer $(\frac{dC}{dt})$ and revenue $(\frac{dK}{dt})$ behaviour.

For consumers $(\frac{dC}{dt})$: $\frac{\partial(dC/dt)}{\partial r} = C$, $\frac{\partial(dC/dt)}{\partial \alpha} = -CK$, $\frac{\partial(dC/dt)}{\partial \delta} = 0$, $\frac{\partial(dC/dt)}{\partial \beta} = 0$

For revenue $(\frac{dK}{dt})$: $\frac{\partial(dK/dt)}{\partial r} = 0$, $\frac{\partial(dK/dt)}{\partial \alpha} = 0$, $\frac{\partial(dC/dt)}{\partial \delta} = -K$, $\frac{\partial(dK/dt)}{\partial \beta} = CK$

These derivatives were evaluated throughout the simulation period (Q31–Q34) to identify the most influential parameters.

Results indicated that:

- Consumer growth is most sensitive to r and α.
- Revenue dynamics are most sensitive to δ and β.

Parameter Estimation

Parameter estimation was performed in two stages:

a. First, Initial Estimation via Nonlinear Least Squares Regression (NLSR) using scipy.optimize.least_squares in Python.

- Numerical derivatives of observed time-series data were calculated using finite differences.
- Parameters $(r, \alpha, \delta, \beta)$ were fitted by minimizing squared errors between observed and model-predicted derivatives.

NLSR Formula:

$$min\theta \sum_{i=1}^{n}(y_i - f(x_i, \theta))^2 \quad (13)$$

Where y_i = observed data and $f(xi,\theta)$ = model-predicted values.

Let $t_1, t_2, \dots, t_n$ be the discrete time points over which data is available. At each time t_i, the observed values of $C(t_i)$, and $K(t_i)$) are used to approximate the derivatives using forward finite differences:

$$\frac{dC}{dt}\Big|_{t_1} \approx \frac{C(t_{i+1})-C(t_i)}{\Delta t} \tag{14}$$

Similarly,

$$\frac{dK}{dt}\Big|_{t_1} \approx \frac{K(t_{i+1})-K(t_i)}{\Delta t} \tag{15}$$

Denote:

$$y_{1,i} = \frac{\Delta C}{\Delta t} \qquad y_{2,i} = \frac{\Delta K}{\Delta t}$$

These approximated derivatives are treated as the dependent variables in the regression framework, while the terms in equations (1) and (2) define the nonlinear functions of the parameters.

At each time point t_i, the residuals for the equations are defined as:

$$\varepsilon_{1,i} = y_{1,i} - [rC(t_i) - \alpha C(t_i)K(t_i)] \tag{16}$$

$$\varepsilon_{2,i} = y_{2,i} - [-\delta K(t_i) + \beta C(t_i)K(t_i)] \tag{17}$$

These residuals represent the deviations between the observed and modelled dynamics at each time point. The NLSR estimation procedure aims to find the parameter vector $(\alpha, \beta, r, \delta)$ that minimizes the sum of squared residuals across all time points:

$$S(\theta) = \sum_{i=1}^{n} (\varepsilon_{1,i}^2 + \varepsilon_{2,i}^2) \tag{18}$$

The optimal parameter vector $\hat{\theta}$ is then obtained by solving: $\hat{\theta} = \arg\min_{\theta} S(\theta)$ using Python's scipy.optimize.least_squares, which implements iterative algorithms (e.g., Levenberg–Marquardt, trust-region methods). It provides initial guesses for $(\alpha, \beta, r, \delta)$ and the optimizer iteratively adjusts parameters to minimize $S(\theta)$.

The baseline (NLSR) parameter estimates are 8.3396, 4.8542×10^{-12}, 5.5475×10^{-17}, and 8.6698×10^{-9} representing r, α, δ and β respectively.

b. To address the unrealistic equilibria produced by the initial NLSR parameter forecast, parameters were refined via Differential Evolution with Logarithmic Transformation. Differential Evolution (Ahmad et al., 2022) is a stochastic global optimization algorithm that searches for a parameter vector θ minimizing a cost function $J(\theta)$ by evolving a population of candidate solutions over multiple generations. Each iteration (or "generation") combines and mutates parameter vectors to explore the search space broadly, while retaining the best-performing solutions. Before optimization, each parameter of the modified Lotka–Volterra model was transformed into log-space (Benoit, 2011): $\phi = log\,(\theta) = (log\ r, log\ \alpha, log\ \delta, log\ \beta)$. This transformation provides three benefits:

- Positivity constraint: ensures all parameters remain positive after exponentiation($\theta = e^{\emptyset}$).
- Scale stabilization: handles large differences in parameter magnitudes($e.g., r \sim 10^{-2}, \alpha \sim 10^{-7}$).
- Smoother optimization landscape: log-scaling often converts a rugged error surface into one more continuous and easier for DE to navigate.

During optimization, DE evolves $\emptyset$, not θ, and converts back using exponentiation for model evaluation. Instead of fitting instantaneous derivatives (as NLSR did), DE minimizes the total squared difference between observed trajectories (consumer and revenue time series) and model-predicted trajectories obtained via RK4 integration of the differential equations.

Formally:

$$J(\emptyset) = \sum_i \left[\left(C_i^{obs} - C_i^{sim}(\emptyset)\right)^2 + \left(K_i^{obs} - K_i^{sim}(\emptyset)\right)^2 \right] \tag{19}$$

where:

- C_i^{sim}, K_i^{sim} are the simulated trajectories using parameters $= e^{\emptyset}$;
- C_i^{obs}, K_i^{obs} are observed from the KEDCO dataset.

This means the optimizer directly tunes parameters to reproduce actual dynamics, not just derivative slopes.

The algorithm for our equation is implemented via the following steps:

i. Initialize population: generate N_p candidate parameter vectors;
$\emptyset_j = (log\ r_j, log\ \alpha_j, log\ \delta_j, log\ \beta_j)$ randomly within defined bounds.

ii. For each target vector $\emptyset_i$, select three distinct random vectors $\emptyset_a$, $\emptyset_b$, $\emptyset_c$ from the population and create a mutant: $v_i = \emptyset_a + F(\emptyset_b - \emptyset_c)$;

iii. **Crossover**: Elements of the mutant vector are combined with the target vector to form a trial vector u_i:

$$u_{i,j} = \begin{cases} v_{i,j}, & if\ rand_j < CK\ or\ j = j_{rand} \\ \emptyset_{i,j}, & otherwise \end{cases} \tag{20}$$

With crossover rate $CK \in [0,1]$.

iv. **Selection**: The trial vector u_i is evaluated using the **objective function** ($J(\emptyset) = \sum_{i=1}^{n} \left[\left(C_i^{obs} - C_i^{sim}(\emptyset)\right)^2 + \left(K_i^{obs} - K_i^{sim}(\emptyset)\right)^2 \right]$). If it produces a lower error than the target vector $\emptyset_i$, it replaces it in the next generation.

v. **Iteration**: These steps are repeated for a fixed number of generations (e.g. 1000) or until the improvement in the objective function falls below a specified threshold.

The parameter refinement was implemented in Python using the scipy.optimize.differential_evolution library. The RK4 integration scheme provided the simulated consumer and revenue trajectories at quarterly intervals.

Initial NLSR estimates served as starting points for Differential Evolution, ensuring a guided yet robust parameter search.
The refined parameters obtained are 4.9638×10^{-5}, 1.4221×10^{-14}, 2.7929×10^{-8} and 2.5662×10^{-8} representing r, α, δ and β respectively.
Unlike the gradient-based NLSR, which often converged to local minima and produced unstable equilibria, the DE method performed a global search of the parameter space. This yielded stable and economically meaningful estimates that reproduced realistic consumer-revenue cycles and equilibrium behaviour.
Overall, the DE with logarithmic transformation process ensured, all parameters remained positive and numerically stable, global convergence to the best-fitting parameter set, and realistic, neutrally stable Lotka-Volterra dynamics consistent with observed KEDCO data.

Numerical Simulation

The coupled nonlinear differential equations of the modified Lotka-Volterra model were solved numerically using the fourth-order Runge-Kutta (RK4) method. This scheme was selected for its balance of computational efficiency, accuracy and stability when applied to nonlinear dynamic systems. RK4 method starts at an initial point $y(t_0) = y_0$, calculates four slopes (intermediate slopes: k_1, k_2, k_3, k_4).
We write the system compactly as a vector ODE

$$\frac{dy}{dt} = f(t, y), \qquad y = \begin{pmatrix} C \\ K \end{pmatrix}$$

$$f(t, y) = \begin{pmatrix} rC - \alpha CK \\ -\delta K + \beta CK \end{pmatrix} \tag{21}$$

The RK4 algorithm approximates the solution at the next time step as follows: Given the state $y_n = y(t_n)$ and $h: step\ size = 1$ (quarterly data), RK4 computes y_{n+1} by:

$k_1 = h * f(t_n, y_n)$: Slope at the beginning.
$k_2 = h * f(t_n + \frac{h}{2}, y_n + \frac{k_1}{2})$: Slope at the midpoint using k_1
$k_3 = h * f(t_n + \frac{h}{2}, y_n + \frac{k_2}{2})$: Another midpoint slope using k_2
$k_4 = h * f(t_n + h, y_n + k_3)$: slope at the end of the interval using k_3
Compute the weighted average of the slopes

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \tag{22}$$

Because y is 2-dimensional, each k_i is a 2-vector. Concretely:

$$k_1 = \begin{pmatrix} rC_n - \alpha C_n K_n \\ -\delta K_n + \beta C_n K_n \end{pmatrix} \tag{23}$$

Compute k_2, k_3, k_4, advance to the next time step and repeat the process until the desired end time.

RESULTS AND DISCUSSION

All simulations, parameter estimation, and data analyses in this study were conducted using the Python programming language due to its versatility in scientific computing.

Forecast Comparison

Two parameter sets, NLSR and Refined, were compared. RK4 simulation was used to solve the equations and forecast Q31 to Q34. NLSR produced extreme divergence, with unstable predictions (including infinities and NaNs (Not a Number)). Conversely, Refined parameters yielded stable and realistic forecasts, with consumer predictions close to observed values (≈2 million) and revenue forecasts in the range of 1.09×10^{10} to 1.28×10^{10}, consistent with empirical magnitudes.

Table 2. Observed vs Predicted Consumers and Revenue (Q31–Q34)

Quarter	Consumers (Observed)	Revenue (Observed)	Consumers (NLSR)	Revenue (NLSR)	Consumers (Refined)	Revenue (Refined)
31	2,037,146	$1.2007e^{10}$	$6.8470e^{8}$	$2.0228e^{10}$	$2.046478e^{6}$	$1.0934e^{10}$
32	2,039,023	$1.2956e^{10}$	$-4.3312e^{12}$	$7.9145e^{15}$	$2.046252e^{6}$	$1.1524e^{10}$
33	2,066,822	$1.4956e^{10}$	$8.7930e^{76}$	$-1.5705e^{80}$	$2.046010e^{6}$	$1.2145e^{10}$
34	2,075,283	$1.3139e^{10}$	∞	NaN	$2.045748e^{6}$	$1.2800e^{10}$

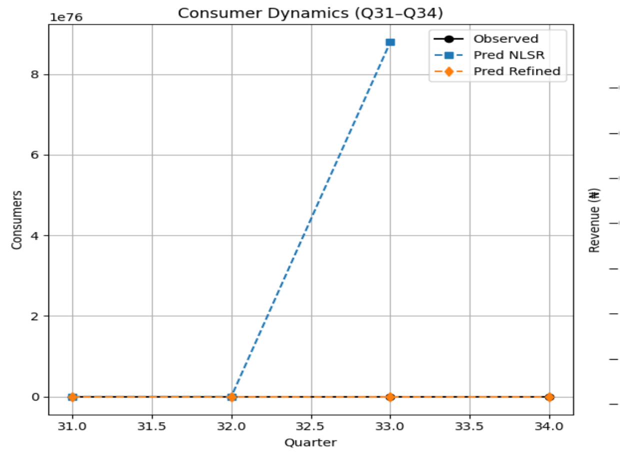

Figure 1. Forecast Trajectories (linear scale)

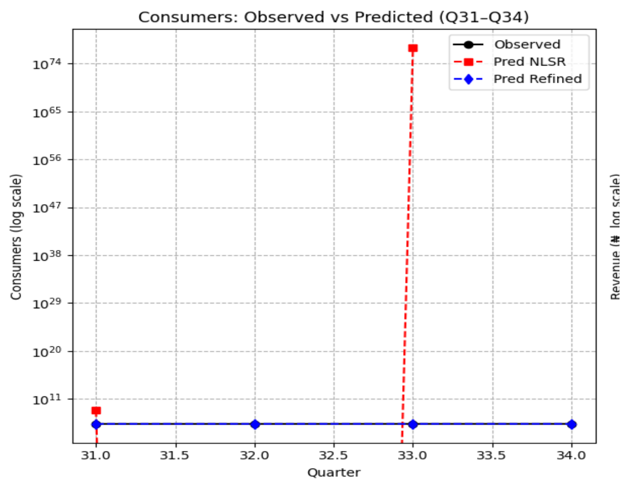

Figure 2. Forecast Trajectories (log scale)

Forecast results demonstrated the inadequacy of NLSR, which diverged numerically, and the superior performance of the refined parameters, which aligned with observed consumer and revenue magnitudes and reliably captured observed dynamics (Q31–Q34), supporting its validity for short-term forecasting in electricity distribution. Logarithmic plots highlighted the NLSR model's catastrophic instability compared to the robust trajectories of the refined model.

Error Metrics

Following the RK4 simulation and comparison of both NLSR and refined parameters forecasts, the forecast accuracy was assessed using RMSE, MAE, and MAPE. These metrics are generally used to quantify how far model predictions deviate from actual values (Scherbakov et al., 2013). They are computed as follows: Let y_i represent the actual value, $\hat{y}_i$ represent the observed/predicted value and n is the number of observations. Then the error $e_i = y_i - \hat{y}_i$ denotes the error at point i.

- MAE: Mean Absolute Error computes the average absolute difference between predictions and actual values. Thus, it treats all errors equally.

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|e_i|$$

- RMSE: Root Mean Square Error first squares errors, takes the average of the squared errors and then takes the square root. It strongly highlights large errors.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i{}^2}$$

- MAPE: Mean Absolute Percentage Error expresses error as a percentage of the actual value.

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{e_i}{y_i}\right| \times 100$$

The error assessment yielded the following result:

- NLSR parameters exhibited infinite error metrics, confirming complete numerical instability;
- Refined model achieved low errors for consumers (MAPE= 0.81%) and reasonable accuracy for revenue (MAPE= 10.34%).

Thus, refined parameters dramatically outperformed the initial estimates.

Table 3. Forecasting Error Metrics (Q31–Q34)

Model	RMSE (Consumers)	MAE (Consumers)	MAPE Consumers(%)	RMSE (Revenue)	MAE (Revenue)	MAPE Revenue(%)
NLSR	∞	∞	∞	$9.0670e^{79}$	$5.2349e^{79}$	$3.5002e^{71}$
Refined	19,005.15	16,726.96	0.81	$1.6743e^{9}$	$1.4133e^{9}$	10.34

Equilibrium Analysis and Phase Planes

After conducting numerical equilibrium analysis, the NLSR equilibrium yielded economically unrealistic values, whereas refined parameters produced a plausible coexistence equilibrium in the billions of Naira range consistent with actual revenue magnitudes.

Table 4. Equilibrium Points and Stability Results

Model	$\boldsymbol{C}^*$(Consumers)	$\boldsymbol{K}^*$ (Revenue)	Eigenvalues	Stability
NLSR	6.40×10^{-9}	1.72×10^{12}	$\pm 2.15 \times 10^{-8} i$	Neutral centre (unrealistic)
Refined	1.088	3.49×10^{9}	$\pm 1.18 \times 10^{-6} i$	Neutral centre (realistic)

- **NLSR Parameters** yielded unrealistic equilibrium values $(C^* \approx 6.4 \times 10^{-9}, \quad K^* \approx 1.7 \times 10^{12})$, suggesting a near-zero consumer base alongside implausibly high revenue.
- **Refined Parameters** produced economically meaningful equilibria $(C^* \approx 1.1, K^* \approx 3.5 \times 10^{9})$, reflecting a plausible balance.

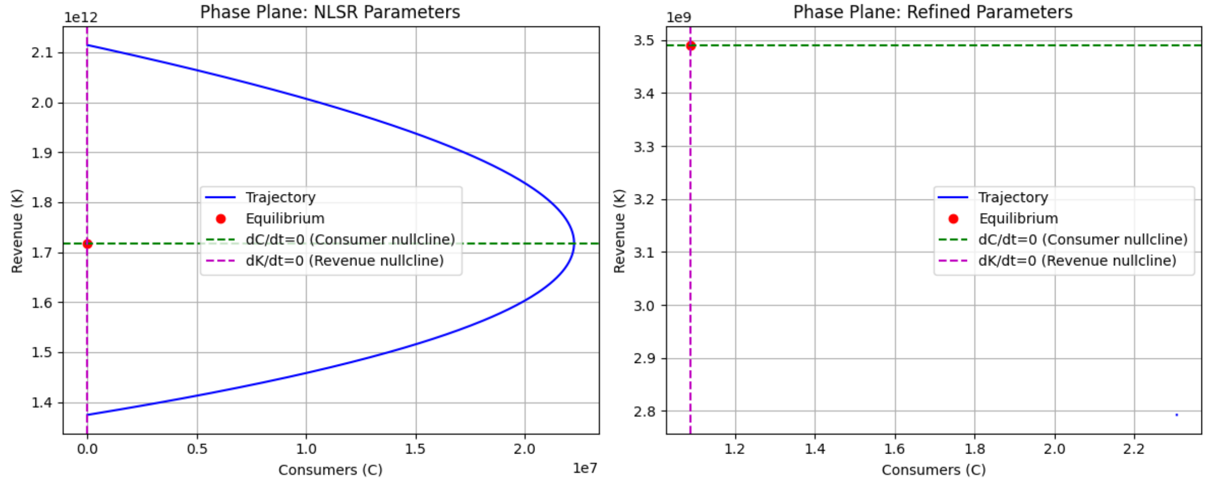

Figure 3. Phase Plane Trajectories

Sensitivity Analysis

Normalized sensitivity analysis confirmed that consumer growth was most sensitive to r and α, while revenue was most sensitive to δ and β. This decomposition provided insight into parameter influence, guiding interpretation and policy applications.

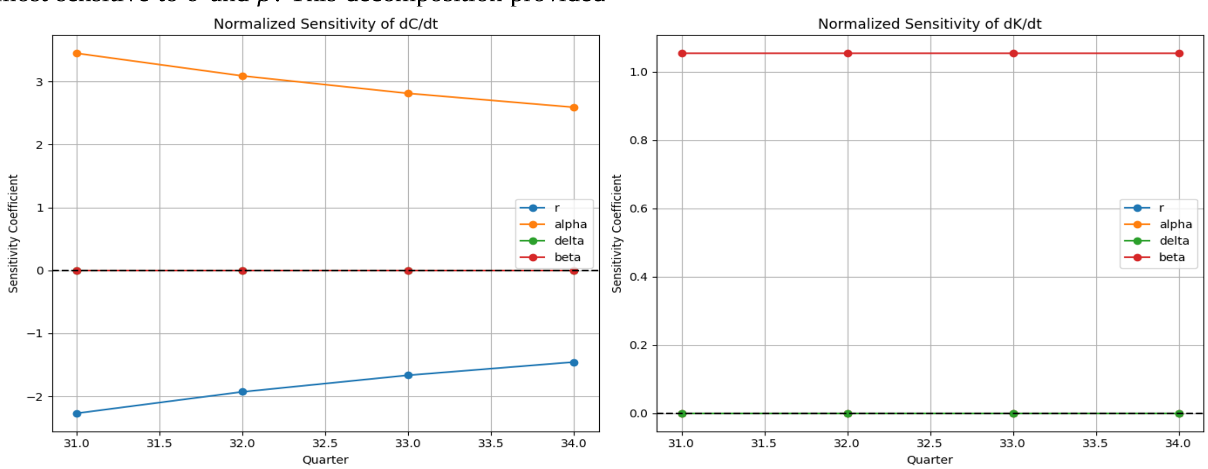

Figure 4. Sensitivity of State Equations to Model Parameters

The results demonstrate the limitations of derivative-based fitting methods such as NLSR when applied to nonlinear, large-scale datasets. Divergence and unrealistic equilibria were observed confirming that it is not suitable for the consumer-revenue system under study. This underscored the need for robust optimization techniques. The use of Differential Evolution with logarithmic scaling produced stable simulations and interpretable equilibria, aligning with both the magnitudes and dynamics of the observed data. Compared to Maku et al. (2023) who applied ARIMA/ARIMAX to forecast Nigerian electricity consumption, their approach emphasized temporal prediction without modelling underlying structural feedbacks. By contrast, the present study captures the cyclical consumer-revenue relationship, adding explanatory depth. Mitropoulou et al. (2022) highlighted predator-prey dynamics in competitive electricity markets, but focused on inter-firm competition in Greece in contrast this study extends that modelling logic to consumer-revenue interactions in Nigeria, demonstrating broader applicability of ecological-inspired models to electricity economics.

CONCLUSION

This study demonstrated the utility of a modified Lotka-Volterra framework for providing valuable insights into consumer-revenue dynamics (and short-term forecasting) within the electricity distribution sector. While initial NLSR estimation produced unstable and economically unrealistic results, refinement using Differential Evolution with logarithmic transformation significantly improved stability, interpretability, and forecasting accuracy that align with real-world KEDCO data. The findings offer a framework that balances predictive capability with interpretability, providing actionable insights for electricity distribution planning in Nigeria. For policymakers and KEDCO, the findings highlight the importance of modelling approaches that incorporate nonlinear dynamics. Beyond forecasting, such models can reveal systemic vulnerabilities and guide interventions such as tariff reforms, loss reduction, and targeted metering strategies.

Recommendations include adopting robust parameter refinement methods, validating models with out-of-sample data, exploring alternative model structures such as time lags and segmentation, and integrating external economic and policy variables for greater realism. Overall, the study provides a policy-informing framework for balancing consumer growth with sustainable revenue recovery, offering a pathway toward improved performance of distribution companies in Nigeria and similar contexts.

REFERENCE

Abe, O. M., Orike, S., & Nkoi, B. (2021). Energy auditing of an electricity distribution system in Nigeria: A case study of Port Harcourt Electricity Distribution Company. *Journal of Newviews in Engineering and Technology (JNET), 3(3).* http://www.rsujnet.org/index.php/publications/2021-edition

Adebayo, A. V. & Ainah, P. K. (2024). Addressing Nigeria's Electricity Challenges: Past, Present, And Future Strategies. *American Journal of Applied Sciences and Engineering, 5(2)* 1-16. https://doi.org/10.5281/zenodo.12633012

Ahmad, M. F., Isa, N. A. M., Lim, W. H., & Ang, K. M. (2022). Differential evolution: A recent review based on state-of-the-art works. *Alexandria Engineering Journal*, *61*, 3831–3872. https://doi.org/10.1016/j.aej.2021.09.013

Amadi, H.N., Okafor E.N.C., Izuegbunam F.I. (2016) Assessment of Energy Losses and Cost Implications in the Nigerian Distribution Network. *American Journal of Electrical and Electronic Engineering*, Vol. 4, No. 5, 123-130. DOI:10.12691/ajeee-4-5-1.

Benoit, K. (2011). *Linear Regression Models with Logarithmic Transformations*. Methodology Institute, London School of Economics. (Technical paper/unpublished manuscript)

Brauer, F., & Castillo-Chavez, C. (2011). *Mathematical models in epidemiology*. Springer. https://doi.org/10.1007/978-1-4419-9745-5

Chaku E.S., Maikatsina B.I., & Umar M.I. (2025). Comparative Analysis of Machine Learning Algorithms for Stock Price Prediction. *Journal of Basics and Applied Sciences Research, 3(6),* 243-251. https://dx.doi.org/10.4314/jobasr.v3i6.24

Edomah N., Ndulue G., Lemaire X. (2021). A review of stakeholders and interventions in Nigeria's electricity sector. Heliyon. https://doi.org/10.1016/j.heliyon.2021.e07956. Volume 7, Issue 9

Efekemo, E., Saturday, E. G., & Ofodu, J. C. (2022). Electricity demand forecasting: A review. *International Journal of Multidisciplinary and Current Educational Research (IJMCER), 4*(2), 279–301.

Hritonenko, N., & Yatsenko, Y. (2010). *Mathematical modeling in economics, ecology and the environment.* Springer. https://doi.org/10.1007/978-0-387-85582-1

Kano Electricity Distribution Company. (n.d.). *About us*. Retrieved April 6, 2025, from https://www.kedco.ng

Lotka, A. J. (1925). *Elements of Physical Biology*. Williams & Wilkins.

Maku, T. O., Adehi, M. U., & Adenomon, M. O. (2023). Modeling and forecasting electricity consumption in Nigeria using ARIMA and ARIMAX time series models. *Science World Journal, 18*(3), 414. https://doi.org/10.4314/swj.v18i3.14

Mitropoulou, P., Papadopoulou, E., Dede, G., & Michalakelis, C. (2022). Forecasting competition in the electricity market of Greece: A prey–predator approach. *SN Operations Research Forum, 3*(41). https://doi.org/10.1007/s43069-022-00143-x

Murray, J. D. (2002). *Mathematical biology: I. An introduction* (3rd ed.). Springer. https://doi.org/10.1007/b98868

Nigerian National Bureau of Statistics. (n.d.). *Historical data on consumer population (prepaid and estimated) and total revenue (Q1 2015–Q2 2023)*. Retrieved March 4, 2025, from https://www.nigerianstat.gov.ng

Shcherbakov, M. V., Brebels, A., Shcherbakova, N. L., Tyukov, A. P., Janovsky, T. A., & Kamaev, V. A. E. (2013). A survey of forecast error measures. *World applied sciences journal*, *24*(24), 171-176.

Volterra, V. (1926). Variazioni e fluttuazioni del numero d’individui in specie animali conviventi. *Memorie dell’Accademia Nazionale dei Lincei*, *Ser. 6*, *2*(3), 31–113.