Journal of Basics and Applied Sciences Research (JOBASR)
ISSN (print): 3026-9091, ISSN (online): 1597-9962
Volume 3(3) May 2025
DOI: https://dx.doi.org/10.4314/jobasr.v3i3.27

Transverse Magnetic Field's Impact on Mixed Convection Flow of an Exothermic Fluid over a Porous Material-filled Channel

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ABSTRACT
This study examines the influence of transverse magnetic field on a steady/constant mixed convection flow of an exothermic fluid down a vertical channel filled with porous material that has symmetric wall temperature and symmetric wall concentration. However in order to achieve that, the Differential Transformation Method (DTM) was used to solve the resulting systems of governing equations analytically, for a variety of physical factors, including magnetic number (M), porous material (Da), mixed convection parameter (λ), constant pressure gradient(γ), Frank-Kamenetskii parameter(K) and sustention parameter (N). After the governing equations were obtained and analyzed, it was concluded from the result that increase in magnetic field can significantly leads to the decrease in the velocity of the fluid. By showing an increase in symmetric wall temperature and concentration, the lower and upper plates also showed the increase in the least and most of the flow velocity, temperature and concentration. The impacts of skin friction, Sherwood number and Nusselt number have also been examined; the graphical result indicates that a decrease in skin friction of the channel wall occurs as the magnetic field number increases.

Keywords:
Mixed convection,
Differential transformation method,
Transverse Magnetic Field,
Exothermic chemical Reaction,
Vertical channel,
Porous material.

INTRODUCTION

A team of scientists, including Pop and his colleagues, worked on a model of a steady fully developed mixed convection flow of a first-order chemical reaction over a vertical channel in 2009. After a numerical analysis of the dimensionless ordinary differential equations of the model they proposed, they discovered that there were two solutions for the fluid's temperature and velocity. Ahmad *et al.* (2017) expanded on Pop *et al.* (2010)'s work a few years later, considering both concentration, temperature and velocity of the fluid, looking at how mass transfer affected the mixed convection flow/transfer of an exothermic fluid over a vertical channel. In view of the aforementioned it is very much evident to states that the problem of heat and mass transfer over a vertical or horizontal porous media of natural, forced or mixed convection, steady or unsteady transfer, symmetrically or asymmetrically heatedin the presence of an exothermic chemical reaction has recently drawn a lot of attention from researchers worldwide due to their practical applications in nuclear reactors, modern agriculture, geothermal reservoir, modern electronics, thermal insulation, energy storage and conservation, as well as petroleum reservoirs. In order to manage the cooling pace and achieve the desired quality of the industrial output, a magnetic field is one of the key components or essential elements needed to control the rate of cooling and provide industrial output of the appropriate quality. The characteristics of mixed convection flow through porous media have been the subject of numerous investigations.

In recent decades, a lot of research has been done on the mixed convection flow of an exothermic chemical reaction. Investigators evaluated a wide range of thermal boundary condition setups and combinations. Rudraiah *et al*., (1995), examined how magnetic fields affected natural convection in a rectangular cage, Chamkha (2003). Studied the behavior of g-jitter induced natural convection in microgravity under the influence of a transverse magnetic field and in the presence of heat generation or absorption effects for a simple system made up of two parallel impermeable infinite plates held at four distinct thermal boundary conditions, Khanafer *et al*., (1999) talked about mixed convection flow in a lid-driven container that contains a porous material that is saturated with fluid. Weng (2005) examined the fully developed free convection flow over an open-ended vertical parallel plate micro-channel analytically while taking the asymmetric wall temperature distribution into account, while Hamza (2016) examined the impact of free/natural convection slip flow of an exothermic fluid over a vertical channel in an effort to better understanding the effects of natural convection on heat and mass transfer flows. Jha *et al*., (2014) also expand on this study by including suction/injection on the micro-channel walls.The effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field has been studied by Afify (2004). Saleh *et al.,* (2013) looked atthe flow reversal of fully developed mixed convection in a vertical channel with chemical reaction, Jha *et al*., (2015) observed and examined MHD free convection flow through amicro-channel over a vertical parallel plates, Hayat *et al.,* (2015) studied mixed convection flow of cassonNanofluid over a stretching sheet with heat source/sink and convectively heated chemical reaction. Moreover, hydro magnetic instability of free convection flow of a heat absorbing fluid within a rotating vertical channel in porous medium with hall effects have been studied by Seth *et al.,* (2016), Srinivas *et al.,* (2010) examined impacts of heat radiation and space porosity on magnetohydro dynamics mixed convection flow over a vertical channel considering homotophy analysis approach. Das and Jana (2010) examined the impact of heat and mass transfer over an unsteady natural convection flowthrough porous media close to a moving vertical plate. Recently the impact of thermal diffusion (soret term) on heat and mass transfer flow across a vertical medium in the presence of magnetic field intensity were studied by Ibrahim *et al.,* (2024).

In this study, the continuous mixed convection flow through a vertical porous medium in the presence of symmetric wall temperature and symmetric wall concentration under the impact of a transverse magnetic field.

MATERIALS AND METHODS

Mathematical Formulation

Figure 1 illustrates how the two vertical porous channel walls are separated by a distance L while maintaining a constant and distinct temperature at the top and bottom T_a and T_m respectively, such that $T_a > T_m$ to account for steady state heat and mass transfer of mixed convection flow of an exothermic fluid between the walls. In order to alignthe **x-**axis withthe gravitational acceleration vector **g**, but in the opposite direction, a coordinate direction was selected. The axes' origins are such that the channel walls are positioned at y = 0 and y = L, respectively. The **y-**axis is orthogonal to the channel walls.At the channel entry, the fluid has a consistent vertical upward stream-wise velocity distribution U_0, and the walls are subjected to a strong horizontal magnetic field B_0. According to Pop *et al*., (2010) and Ahmad *et al*., (2017), it was assumed that the working fluid was Newtonian, incompressible and flow steadily, it was also hypothesized thatan exothermic surface reaction within the porous channel provided the heat to the surrounding fluid. Thus, we have the following dimensional representations of the Momentum, heat and mass transfer equations based on the previously indicated assumptions;

$$v\frac{d^2u'}{dy'^2}-\frac{\nabla B_0^2u'}{\rho}-\frac{vu'}{k'}+g\beta(T'-T'_0)+g\beta^*(C'-C'_0)-\frac{1}{\rho}\frac{dP'}{dx'}=0 \tag{1}$$

$$\alpha\frac{d^2T'}{dy'^2}+QK_0ae^{-E/RT'}=0 \tag{2}$$

$$D\frac{d^2C'}{dy'^2}=0 \tag{3}$$

Kinematic viscosity, the gravity and the density of the fluid respectively are denoted by v, g, and ρ, Q is the exothermic factor, β is the fluid thermal expansion coefficient, B_0 is the magnetic induction coefficient, β^* is the coefficient of fluid concentration expansion, T_0 is the reference temperature, C_0 is the reference concentration, and assumed that $T_0=(T_a+T_m)/2$ and $C_0=(C_a+C_m)/2$.

Theproblem's boundary conditionsare expressed as follows;

$$u=0,\ T=T'_a,\ C=C'_a,\ y'=0$$
$$u=0,\ T=T'_m,\ C=C'_m,\ y'=L \tag{4}$$

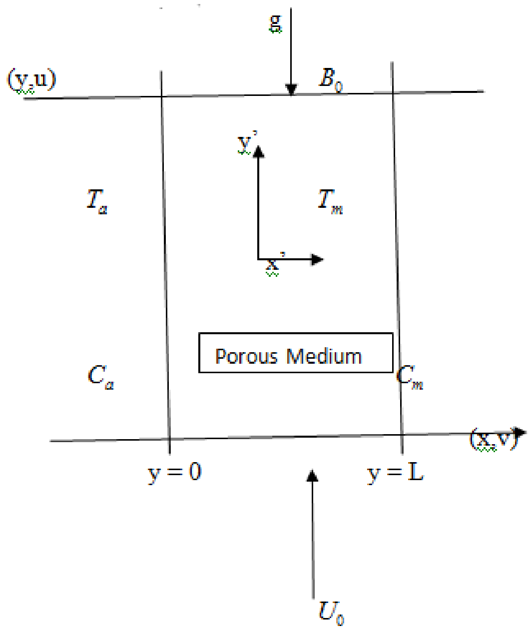

Figure 1. Geometric of the Problem

After that, the following dimensionless variables were introduced to equation (1) through (3) to convert them to dimensionless ordinary differential equations.

$$x=\frac{x'}{\mathrm{Re}\,L},\ \ y=\frac{y'}{L},\ u=\frac{u'}{u_0},\ p=\frac{p'}{\rho u_0^2},$$

$$\mathrm{Re}=\frac{u_0L}{v}\quad Gr=\frac{g\beta RT_0^2L^3}{Ev^2},\ \theta=\frac{T-T'_0}{RT_0^2/E},$$

$$\phi=\frac{C-C'_0}{RC_0^2/E},\ M^2=\frac{\nabla B_0L^2}{\rho v} \tag{5}$$

When equation (5) is substituted in to equations (1) through (4), the following ordinary differential equations result:

$$\frac{d^2u}{dy^2}-\left(M^2+\frac{1}{Da}\right)u+\lambda[\theta+N\phi]=\gamma \tag{6}$$

$$\frac{d^2\theta}{dy^2}+Kre^{\theta}=0 \tag{7}$$

$$\frac{d^2\phi}{dy^2}=0 \tag{8}$$

Subject to the following boundary conditions

$$u=0,\ \theta=\gamma_t,\ \phi=\gamma_c\ \ at\ \ y=0$$
$$u=0,\ \theta=-\gamma_t,\quad \phi=-\gamma_c\ \ at\ \ y=1 \tag{9}$$

Where λ, N, γ, K, Da, M, γ_c, and γ_t, stand for mixed convection parameter, sustention parameter, pressure term, Frank-Kamenetskii parameter, porous material, magnetic field, symmetric wall concentration and symmetric wall temperature respectively, which are described as:

$$\lambda=\frac{Gr}{\mathrm{Re}},\ \gamma=\frac{dp'}{dx'},\ \gamma_t=\frac{T_a'-T_0'}{RT_0^2/E},\ \gamma_c=\frac{C_a'-C_0'}{RC_0^2/E},$$
$$Kr=\frac{EQK_0aL^2}{RT_0^2\alpha}e^{-E/RT_0} \tag{10}$$

Method of Solution

Using the Differential Transformation Method (DTM), equations (6) through (8) subject to (9) were solved semi-analytically; the velocity, temperature and concentrationexpressions are shown below:

$$U(y)=Ey+G(2)y^2+\left(\frac{\alpha E+\gamma-\lambda[B+NA]}{6}\right)y^3+\left(\frac{\alpha G(2)+\gamma-\lambda F(2)}{12}\right)y^4 \tag{11}$$

$$\theta(y)=\gamma_t+By-\frac{Kr}{2}y^2-\frac{Kr}{6}y^3-\frac{Kr}{24}y^4 \tag{12}$$

$$\phi(y)=\gamma_c+Ay \tag{13}$$

Skin Friction at lower plate y = 0 can be expressed as follows;

$$\frac{du}{dy}|_{y=0}=E \tag{14}$$

Skin Friction at upper plate y = 1 can be expressed as follows;

$$\frac{du}{dy}|_{y=1}=E+2G(2)+\frac{\alpha E+\gamma-\lambda[B+NA]}{2}+\frac{\alpha G(2)+\gamma-\lambda F(2)}{3} \tag{15}$$

Moreover the expressions of Nusselt number and Sherwood number at the lower plate y = 0 and upper plate y = 1 are illustrated as follows;

Nusselt number at y = 0.

$$\frac{d\theta}{dy}|_{y=0} = B \tag{16}$$

Nusselt number at y = 1;

$$\frac{d\theta}{dy}|_{y=1} = B - Kr - \frac{Kr}{2} - \frac{Kr}{6} - \frac{Kr}{24} \tag{17}$$

Sherwood number at y = 0.

$$\frac{d\phi}{dy}|_{y=0} = -2\gamma_c \tag{18}$$

Sherwood number at the upper plate y = 1;

$$\frac{d\phi}{dy}|_{y=1} = -2\gamma_c \tag{19}$$

Where E, A, B, G(2), and the remaining fundamental terms used in Differential Transformation Method are defined in appendix B.

RESULTS AND DISCUSSION

The Steady state of coupled heat and mass transfer has been taken in to consideration when studying the impact of Transverse Magnetic field on mixed convection flow of an exothermic fluid over a channel filled with porous material. The governing equations were solved using differential transformation method, which is a semi-analytical method. MatLab programming software has been used to carry out the computation and analyzed the result graphically. Figure 2 is the concentration field profile with varying values of (γ_c) which makes it evident that the fluid's concentrationat y = 0, increases as the value of γ_c increases, however the reverse behavior was seen at the top plate (y = 1).

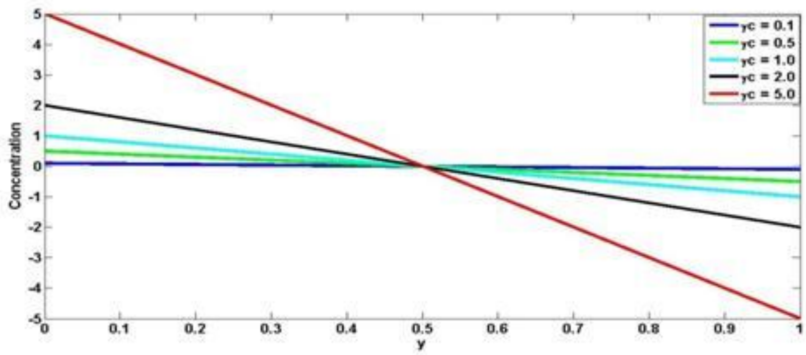

Figure 2. Concentration profile with different values of γ_c, λ = 100, γ = 0.01, K= 1.5

The impacts of symmetric wall temperature (γ_t) on velocity profile is shown in Figure 3, and it can be inferred from the figure that the increase in symmetric wall temperature can effectively increases the velocity field atboth lower and upper plate. It can be seen from Figure 4, which displays the temperature field profile with the effects of different values of (γ_t), that increasing the values of (γ_t) can result in a noticeably higher temperature field. As expected, since heating of the channel wall and the strength of the reaction both significantly rise with an increase in mixed convection parameter and symmetric wall temperature.

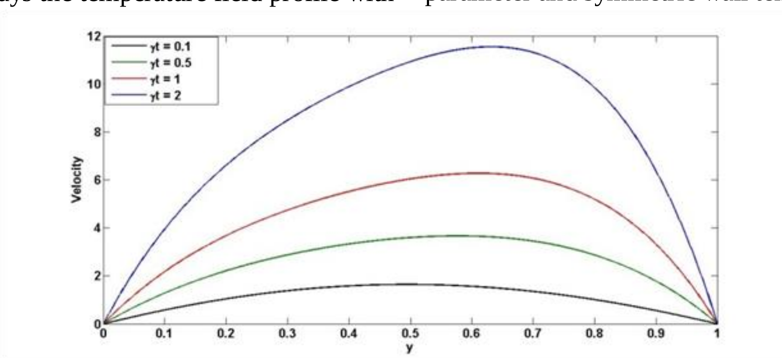

Figure 3. Velocity profile showing Da = 0.1, M = 1, N = 0.01, λ = 100, γ = 0.01, K = 1.5 with different values of γ_t

The effect of Frank-Kamenetskii parameter (K) on temperature profile Figure 5 and it is evident that the fluid's temperature increases as (K) increases. The impact of Darcy number on velocity field was demonstrated in Figure 6, which shows that the velocity increases significantly as the value of Darcy number (Da) increases.

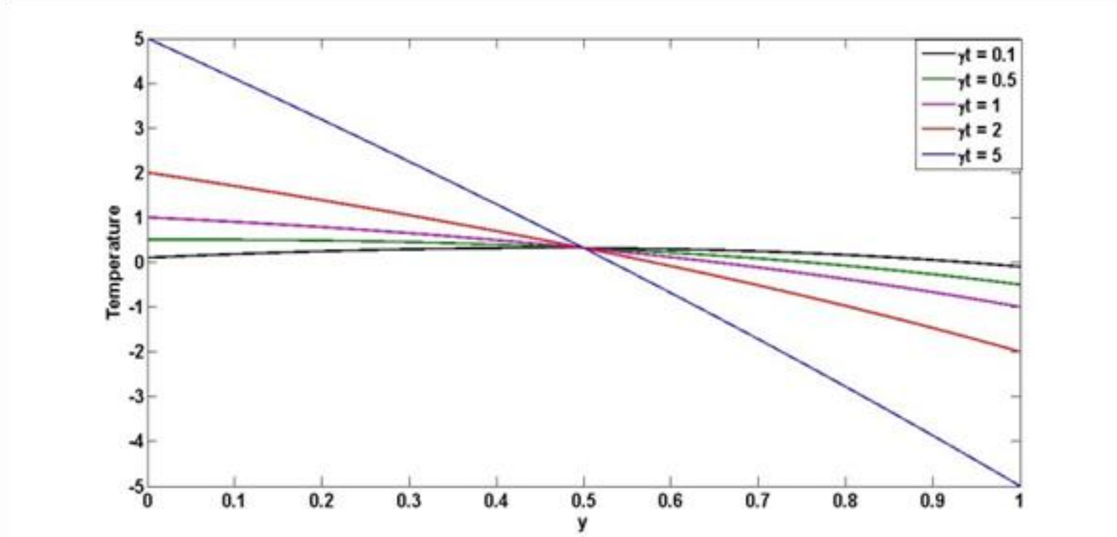

Figure 4. Temperature profile showing Da = 0.1, M = 1, N = 0.01, λ = 100, γ = 0.01, K = 1.5 with different values of γ_t

The relationship between Frank-Kamenetskii parameter (K) and velocity is shown in Figure 7, which indicates that an increase in the value of K can result in an increase in the velocity field. Figure 8 illustrates how magnetic number affects velocity field, it is very clear from the figure that a higher magnetic number (M) can result in a lower fluid velocity.

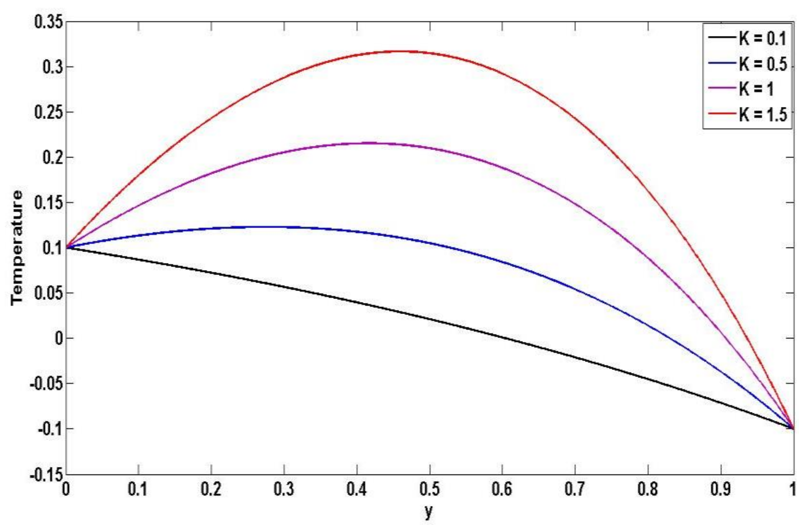

Figure 5. Temperature profile displays Da = 0.1, M = 1, N = 0.01, λ = 100, γ = 0.01, with different values of K.

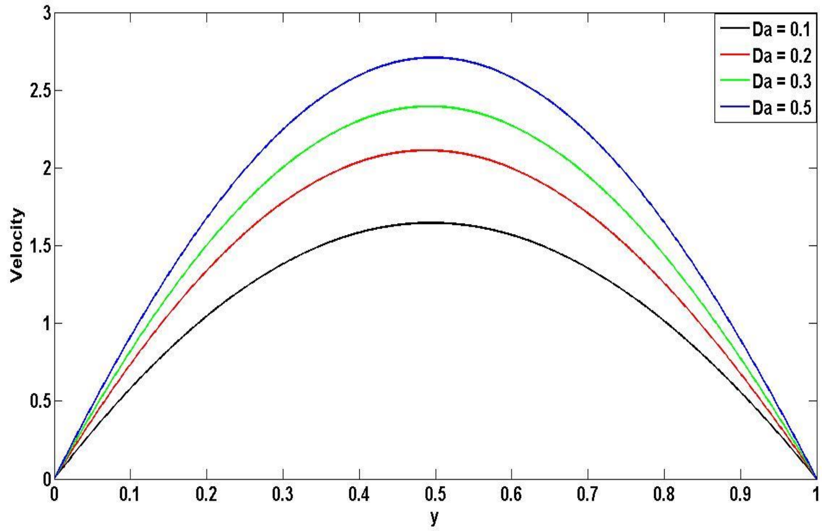

Figure 6. Profile of Velocity displaying M = 1, N = 0.01, λ = 100, γ = 0.01, K = 1.5 and varying values of Da.

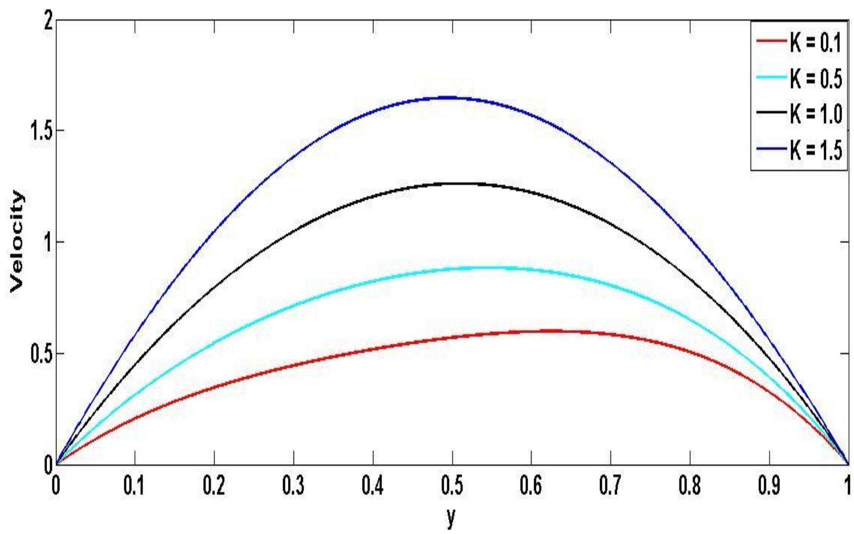

Figure 7. Illustrating how velocity is affected by the Frank-Kamenetskii parameter (K).

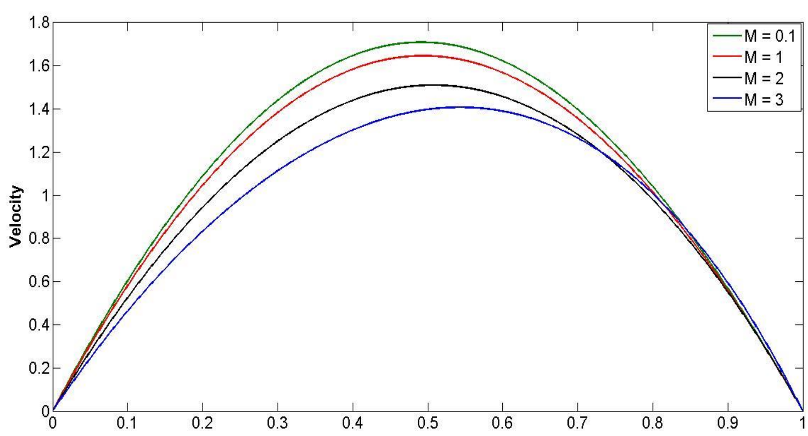

Figure 8. Illustrate the impact of magnetic number (M) on Velocity profile.

The impact of Magnetic number on the Skin friction rate is shown in Figures 9 (a) and 9 (b). Figures 10(a) and 10(b) shows the effect of γ_t on skin friction at the lower plate (y = 0) and at the upper plate (y = 1). Figures 11 (a) and 11(b) illustrate how symmetric wall concentration γ_c affects skin friction.

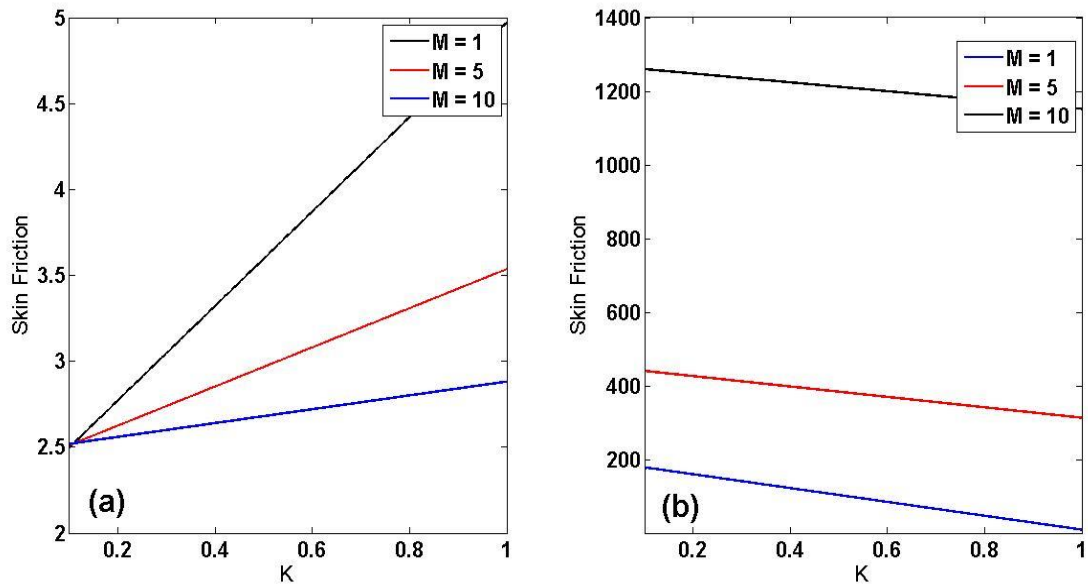

Figure 9. Skin Friction Variation with Magnetic number.

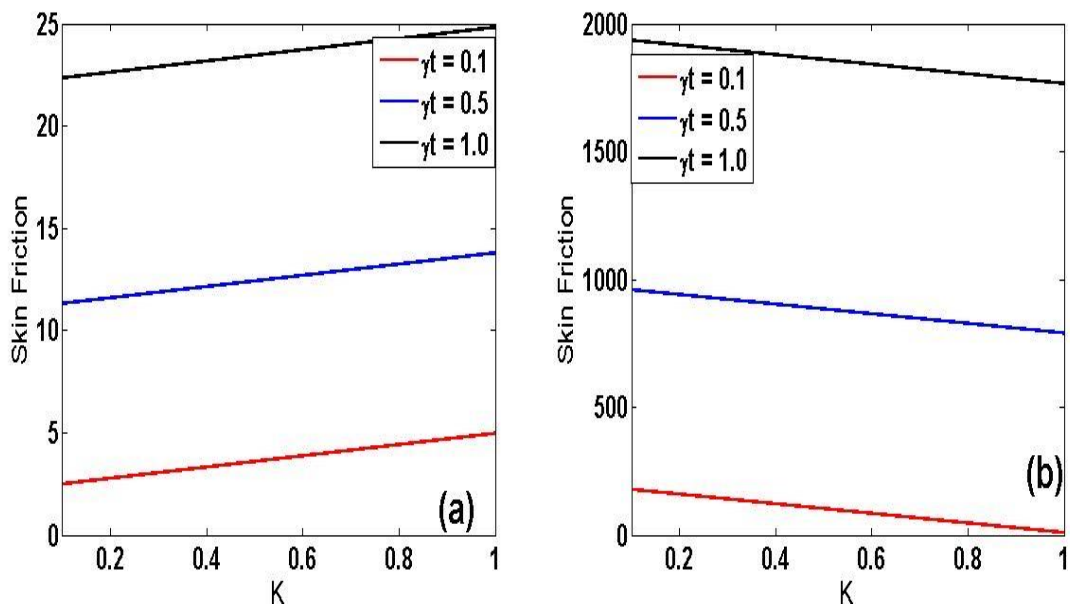

Figure 10. Variation of γ_t on Skin Friction.

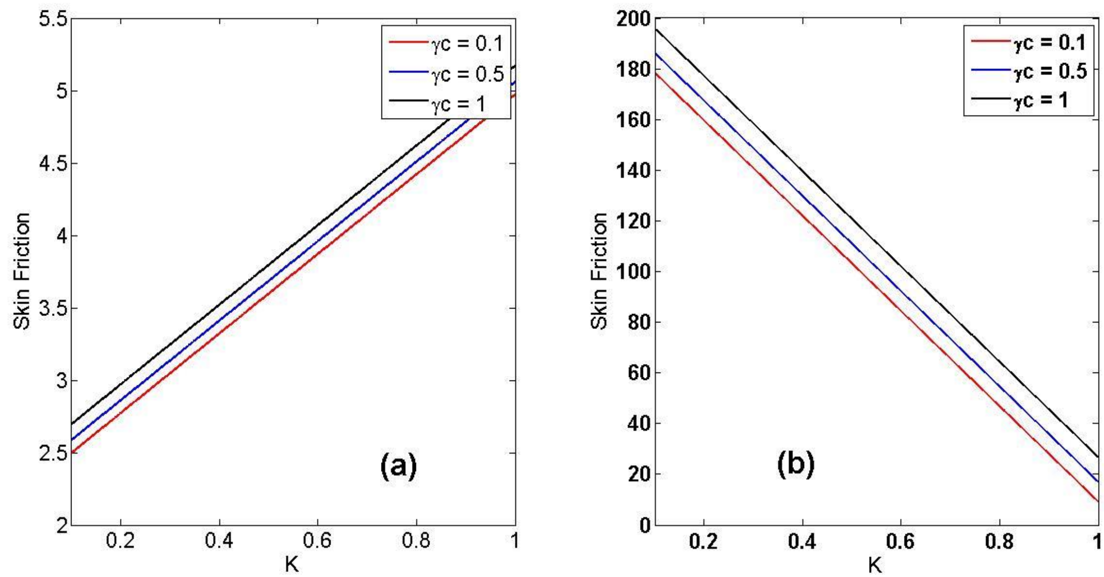

Figure 11. Variation of γ_c on Skin Friction.

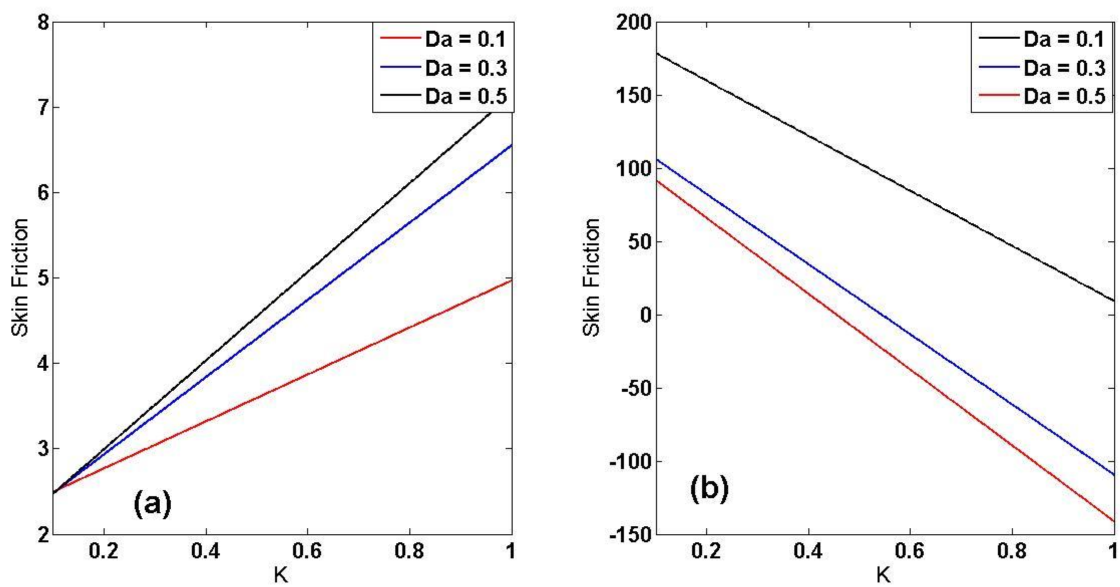

Figure 12. Illustrating how Skin Friction can be affected by the Darcy number.

The impacts of porous medium on skin friction is illustrated in Figures 12 (a) and (b), and it is evident from the figure that an increase in the Darcy number (Da), can accelerate the rate of Skin Friction. In Figures 13 (a) and 13(b), the effects of various values of γ_t on Nusselt number were displayedand it is clear from the graph thatthe rate of heat transfer increases whenever the value of γ_t increases.

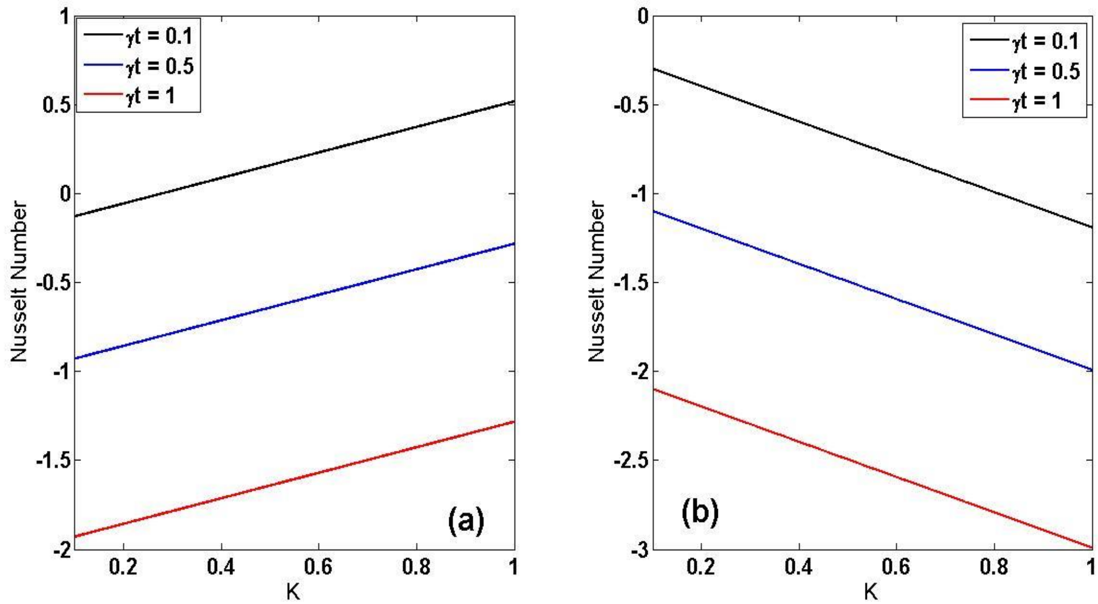

Figure 13. Variation of γ_t on Nusselt number

CONCLUSION

The study examined the impacts of transverse magnetic field on the mixed convection flow of an exothermic fluid in a channel filled with porous material, the coupled solution of the governing equations were obtained usingthe Differential Transformation Method. After taking in to account the influences of magnetic field parameter (M) on velocity and skin friction, the following conclusions and recommendation were drawn from the result.

(i) That the velocity of the fluidrises with increase in Darcy number (Da) and precipitously decreases with an increase in the value of magnetic number (M).

(ii) Skin friction significantly decreased at the lower plate (y = 0) as the value of magnetic number (M), whereas the opposite behavior was seen at the upper plate (y = 1).

(iii) Both temperature and velocity of the fluid increases with increases in value of the Frank-Kamenetskii parameter as shown in figure 5 and 7.

The solutions presented in this study will be of great significance for various thermo physical impacts for subsequent researches both numerically and analytically in heat and mass transfer in modeling of metallurgical transport and many geophysical processes.

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Appendix A

Nomenclatures

The various parameters that regulate the transfer/flow of the fluid in dimensional form are as follows:

Da	: Porous substance
θ	: Dimensionless Temperature
U	: Represents the fluid's dimensionless velocity
y	: Is the co-ordinate without dimensions that is perpendicular to the plate
x	: Is the Dimensionless co-ordinate parallel to the plate
N	: Is the sustaining parameter
K	: Frank-Kamenetskii parameter
λ	: Mixed convection parameter
M	: Stands for Magnetic field number
γ	: Constant pressure gradient
γ_t	: Wall temperature
γ_c	: Wall concentration

Appendix B

A = $-2\gamma_c$ B = $\frac{K}{2}+\frac{K}{6}+\frac{K}{24}+\frac{K}{120}-2\gamma_t$ $\alpha = M^2+\frac{1}{Da}$

$$G(2)=\frac{\gamma-\lambda\left[\gamma_t+N\gamma_c\right]}{2}$$

$$E=\frac{2\lambda(B+NA)-12G(2)-2\gamma-\alpha G(2)-\gamma+\lambda F(2)}{12+2\alpha}$$

$$F(2)=\frac{-K}{2}$$