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A Novel Parsimonious Gamma Mixture with Applications to Reliability Data

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ABSTRACT
Lifetime distributions are salient statistical tools to model the different characteristics of lifetime datasets. The statistical literature contains very modern distributions to analyze these kinds of datasets. Nonetheless, these distributions have many parameters, which cause a problem in the estimation step. To offer fresh possibilities in modeling these kinds of datasets, we propose a Parsimonious Gamma Mixture (PGM) distribution using a finite mixture of Gamma distributions with parameter-dependent mixing weights. The proposed distribution has only one parameter and simple mathematical forms. The mathematical properties of the distributions, including moments, reliability functions and order statistics, are studied in detail. The unknown model parameter is estimated by using the maximum likelihood. The extensive simulation study is used to study the performance of parameter estimation. To convince the readers in favour of the proposed distribution, three real datasets from engineering and materials science are analyzed and compared with competitive models. Empirical findings show that the proposed one-parameter lifetime distribution produces better results than the other similar existing distributions. Its consistent achievement of the lowest AIC and BIC values across all datasets confirms its enhanced ability to capture diverse data patterns with remarkable parameter parsimony, making it a highly effective tool for modelling reliability and survival data.

Keywords:
Parsimonious gamma
Mixture distribution;
Reliability functions;
Model selection
Criteria;
Reliability analysis;
Survival data modelling

INTRODUCTION

The statistical analysis and modelling of lifetime data, which is often referred to as survival or failure time analysis, form a cornerstone of research across a diverse areas of applied sciences: from the failure of mechanical components in engineering to the survival of patients in medical studies and from the time-to-default in financial instruments to the duration of unemployment in economics, the need to accurately model the time-to-event is ever-present (Johnson et al., 1995; Mazucheli et al., 2018; Aderoju, 2021; Aderoju et al., 2025a; Aderoju et al., 2025b). The cardinal objective in these fields is not merely to describe data but to understand underlying failure mechanisms, evaluate reliability and inform critical decision-making processes.

For decades, the classical exponential distribution, with its memoryless property and constant hazard rate, served as a foundational model due to its mathematical simplicity. However, its assumption of a constant failure rate is often a severe limitation when modelling real-world phenomena, where components age or systems improve over time. Recognizing this, Lindley (1958) introduced a distribution that offered more flexibility. This inspired a renewed interest in developing more adaptable and flexible lifetime models. As noted by Shanker et al. (2015), the exponential and classical Lindley distributions are often unsuitable for many real-life datasets due to their restrictive shapes and hazard rate functionalities.

This inadequacy has generated a significant movement in statistical literature towards the creation of more flexible probability distributions. A prominent strategy involves the generalization of existing baseline models, often by adding one or more shape parameters to enhance flexibility (Marshall & Olkin, 2007; Aderoju, 2021). While effective, this approach can lead to complex models with challenging parameter estimation. In contrast, the pursuit of parsimonious models remains a central principle of statistical modelling. As eloquently stated by one source, "the parsimony rule says the best model is a model that requires fewer assumptions and/or parameters"

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(Burnham & Anderson, 2002; Box & Jenkins, 1976). This has driven the development of flexible one-parameter distributions that strive to balance simplicity with the flexibility needed to model complex data characteristics.

The methodology for creating these new one-parameter distributions varies and is innovative. A common method is finite mixture modelling, where a new distribution is formed as a weighted combination of existing distributions, such as the exponential and gamma distributions. For instance, Ghitany et al. (2008) introduced the Lindley distribution by mixing exponential and gamma components. In line with this approach, researchers have proposed several models, such as the Samade (Aderoju, 2021), Power Hamza (Aderoju and Jolayemi, 2022), Ishita (Shanker & Shukla, 2017) and Pranav (Shukla, 2018) distributions, Inverse Power Rama (Chrisogonus et al., 2020), Inverse Power Ishita (Shukla, 2021; Frederick et al., 2022) and Inverse Power Hamza distributions (Frank et al., 2023; Omoruyi et al., 2023). Other significant contributions through this paradigm are the Power Generalized Akash (Aderoju & Adeniyi, 2022), Power Hamza distribution (Aderoju & Jolayemi, 2022), the New Generalized Gamma-Weibull Distribution (Aleshinloye et al., 2023) and a novel variant of the Rama distribution (Omoruyi et al., 2023). This line of work also includes more recent contributions like the New Extended Exponential-Gamma Distribution (Aderoju et al., 2025b) and the new two-parameter generalized Lindley distribution (Aderoju et al., 2025a). Each of these proposed models features distinct mixing proportions and, consequently, unique resulting properties. Other methods include transformations of existing distributions, such as the unit-Lindley distribution for data on the interval (0, 1) and the use of generator families, like the odd Lindley-G family, to introduce new flexibility with a single additional parameter.

The advantages of one-parameter distributions are numerous. They enable easier parameter estimation, especially with smaller sample sizes and their mathematical properties, such as moments, quantile functions and hazard rates, are often more manageable. Moreover, a simple formula for the mean allows for the direct incorporation of covariates to model their average effect on the response variable, allow the development of simplified regression models as alternatives to more complex counterparts like the Beta regression model (Ferrari & Cribari-Neto, 2004).

In this dynamic context, the need for comparative studies is essential. With many one-parameter distributions now available, practitioners and researchers require clear guidance on their relative performance. As different data "sing their song," the quest to find the most suitable probability distribution to capture all its variations is of crucial (Eliwa et al., 2021). The primary motivation for developing new distributions is often the hope that they will provide a superior fit for types of data, achieve minimal error in forecasting and outperform existing models.

In this paper, we introduce a novel one-parameter lifetime distribution that adheres to the principle of parsimony: the Parsimonious Gamma Mixture (PGM) Distribution. The remaining sections of this study are organized as follows. in Section 2, we **derive the new model probability distribution function**. Its key mathematical properties are presented in Section 3. Section 4 includes the maximum likelihood estimation procedure **and assessment of its performance through** a simulation study. Application of the model to real-life data is presented in Section 5. Finally, the concluding remarks are presented in Section 6.

MATERIALS AND METHODS

A novel continuous distribution, the **Parsimonious Gamma Mixture (PGM)**, is presented in this section. The derivation yields its probability density function, establishing the PGM as a mixture of two gamma distributions with a common rate parameter $\beta > 0$. Specifically, it combines Gamma $(2, \beta)$ and Gamma $(3, \beta)$ components, with the mixing proportion functionally dependent on β itself. The mixture distribution is of the form:

$$f(x) = p_1 g_1(x) + p_2 g_2(x), \qquad (1)$$

where

$$g_1(x) \sim \text{Gamma}(2, \beta)$$

$$g_1(x) = \frac{\beta^2}{\Gamma(2)} x e^{-\beta x} = \beta^2 x e^{-\beta x}, \quad x, \beta > 0$$

$$g_2(x) \sim \text{Gamma}(3, \beta)$$

$$g_2(x) = \frac{\beta^3}{\Gamma(3)} x^2 e^{-\beta x} = \frac{\beta^3}{2} x^2 e^{-\beta x}, \quad x, \beta > 0$$

and

$$p_1 = \frac{\beta}{1+\beta}$$

Where $p_1 + p_2 = 1$

Therefore, (1) becomes

$$f(x) = \frac{\beta^3}{2(1+\beta)} x(x+2) e^{-\beta x}$$

Note: This is a valid PDF because:

a) It is non-negative for $x > 0, \beta > 0$.

b) It integrates to one (since it is a mixture of two proper PDFs with weights summing to 1).

c) The weight p_1 represents the probability that an observation is generated from the first component, Gamma (2, β). As the rate parameter β increases, the mean of both component distributions $\left(\frac{2}{\beta} \text{ and } \frac{3}{\beta}\right)$ decreases. The functional form $p_1(\beta)$ ensures that as the distribution shifts towards smaller values (higher β), the weight p_1 also increases. This creates a coherent system where a single parameter β controls both the scale *and* the mixture composition of the distribution.

This link ensures that p_1 is automatically bounded between 0 and 1 for all $\beta > 0$, satisfying the fundamental requirement for a probability weight. Specifically:

$$\lim_{\beta \to 0} p_1(\beta) = 0 \ \ and \ \ \lim_{\beta \to \infty} p_1(\beta) = 1$$

The PGM distribution emerges as a carefully constructed one-parameter model derived from a finite mixture of two Gamma components with shared scale parameters and fixed shape parameters. This strategic parameterization achieves an optimal balance between model simplicity and distributional flexibility, enabling it to capture diverse data patterns, including both decreasing and unimodal hazard rates, while maintaining computational compliance. Its closed-form cumulative distribution function and moments, with its interpretable single parameter that governs tail behaviour, make the PGM particularly well-suited for reliability engineering and lifetime data analysis, where avoiding overparameterization without sacrificing fit quality is key.

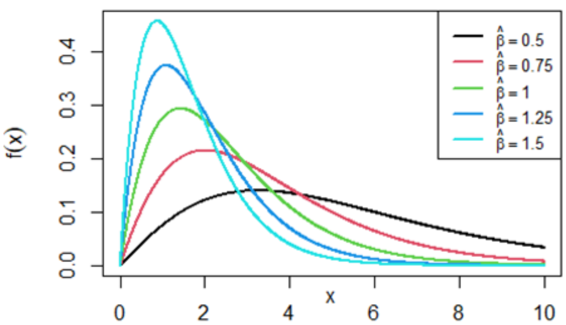

Figure 1: The PDF plots of the PGM Distribution for Different Values of $\hat{\beta}$

Figure 1 shows the probability density function $f(x)$ of the PGM distribution for different values of the parameter's estimate. The curves are all unimodal, meaning they rise from zero, reach a peak and then decline exponentially as x increases. This behaviour reflects how the parameter β controls the rate of decay and the concentration of the density.

The corresponding cumulative distribution function (CDF) is

$$F(x) = 1 - \frac{\left(\beta^2 x^2 + 2\beta x(\beta+1) + 2(1+\beta+\beta^2)\right)e^{-\beta x}}{2(1+\beta)} \quad (2)$$

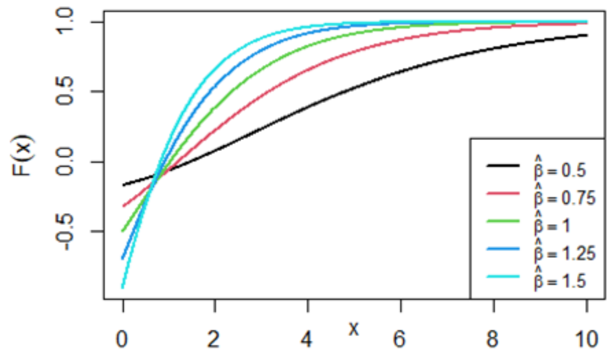

Figure 2: The plots of the PGM Distribution's CDF for various values of the parameter

Figure 2 represents the $F(x)$ plots, which increases from 0 to 1 as x increases. For all values of $\hat{\beta}$, the CDFs rise at different rates. Larger values of $\hat{\beta}$, make the CDF rise more steeply, meaning probabilities accumulate faster and most of the distribution lies near smaller x. Conversely, smaller $\hat{\beta}$ values yield a slower rise in the CDF, which implies heavier tails and a greater chance of larger x. This matches the suspicion from the PDF plot: lower β stretches the distribution, while higher β compresses it.

Mathematical properties of the PGM distribution

This subsection presents the fundamental mathematical properties of the new distribution, which are required for understanding its behaviour and execute statistical inference.

Let $X \sim PGM(\beta)$, then the rth raw moment is given by:

$$E(X^r) = \mu_r = \int_0^\infty x^r f(x) dx$$

$$= \frac{\beta^3}{2(1+\beta)} \int_0^\infty x^{r+1}(x+2)e^{-\beta x} dx$$

$$\mu_r = \frac{\beta^{-r}(2\beta\Gamma(2+r) + \Gamma(3+r))}{2(1+\beta)} \quad (3)$$

Substituting $r = 1, 2, 3, 4$ into equation (3) provides the first four raw moments as:

$$\mu_1 = \frac{3+2\beta}{\beta+\beta^2}$$

$$\mu_2 = \frac{6(2+\beta)}{\beta^2(1+\beta)}$$

$$\mu_3 = \frac{60+24\beta}{\beta^3+\beta^4}$$

$$\mu_4 = \frac{120(3+\beta)}{\beta^4(1+\beta)}$$

The variance of X, coefficient of variation (CV), skewness and kurtosis are derived as follows:

$$\sigma^2 = \frac{3+6\beta+2\beta^2}{\beta^2(1+\beta)^2}$$

$$CV = \frac{\sqrt{(3+6\beta+2\beta^2)}}{\beta(1+\beta)(3+2\beta)}$$

$$Sk = \frac{60+24\beta}{\beta^3(1+\beta)(\frac{3+6\beta+2\beta^2}{\beta^2(1+\beta)^2})^{3/2}}$$

$$Ks = \frac{120(1+\beta)^3(3+\beta)}{(3+6\beta+2\beta^2)^2}$$

Rényi entropy

Rényi entropy measures the uncertainty in a probability distribution. For the PGM distribution, the Rényi entropy of order γ (where $\gamma > 0$ and $\gamma \neq 1$) is given by:

$$H_\gamma(X) = \frac{1}{1-\gamma}\log\left(\int_0^\infty f^\gamma(x)dx\right)$$

$$H_\gamma(X) = \frac{1}{1-\gamma}\log\left(\left(\frac{\beta^3}{2(1+\beta)}\right)^\gamma \int_0^\infty x^\gamma(x+2)^\gamma e^{-\gamma\beta x}dx\right)$$

$$H_\gamma(X) = \frac{1}{1-\gamma}\log\left(\left(\frac{\beta^3}{2(1+\beta)}\right)^\gamma (\gamma\beta)^{-(2\gamma+1)} \sum_{k=0}^{\infty}\binom{\gamma}{k}(2\gamma\beta)^k\Gamma(2\gamma-k+1)\right) \quad (4)$$

Order statistics

Order statistics provide a fundamental framework for inference in reliability engineering and survival analysis. For a random sample $X_1, X_2, \dots, X_n$ drawn independently from the PGM distribution, the extreme observations are defined as $X_{(n)} = \max(X_1, \dots, X_n)$ and $X_{(1)} = \min(X_1, \dots, X_n)$, representing the maximum and minimum values, respectively. The ordered sequence $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$ represents the sorted observations. The probability density function of the k^th order statistic $X_{(k)}$ for the PGM distribution is given by:

$$f_{X_{(k)}}(x) = \frac{n!}{(k-1)!\,(n-k)!} f_X(x)[F_X(x)]^{k-1}[1 - F_X(x)]^{n-k} \quad (5)$$

Substituting (1) and (2) into equation (5), the PDF of $X_{(k)}$ is given as the following:

$$f_{X_{(k)}}(x) = \frac{n!\,\beta^3 x(x+2)}{(k-1)!\,(n-k)!\cdot 2(1+\beta)}\left[1 - \frac{D(x)e^{-\beta x}}{2(1+\beta)}\right]^{k-1}\left[\frac{e^{-x\beta}(2+2(1+x)\beta+x(2+x)\beta^2)}{2(1+\beta)}\right]^{n-k} e^{-\beta x}, \quad (6)$$

Where:
$D(x) = \left(\beta^2x^2 + 2\beta x(\beta+1) + 2(1+\beta+\beta^2)\right)$

Now, the PDF of $X_{(1)}$ and $X_{(n)}$ respectively are given by :

$$f_{X_{(1)}}(x) = \frac{n!\,\beta^3 x(x+2)}{(k-1)!\,(n-k)!\cdot 2(1+\beta)} e^{-\beta x}\left[\frac{e^{-x\beta}(2+2(1+x)\beta+x(2+x)\beta^2)}{2(1+\beta)}\right]^{n-1} \quad (7)$$

and

$$f_{X_{(n)}}(x) = \frac{n!\,\beta^3 x(x+2)}{(k-1)!\,(n-k)!\cdot 2(1+\beta)} e^{-\beta x}\left[1 - \frac{D(x)e^{-\beta x}}{2(1+\beta)}\right]^{n-1} \quad (8)$$

Reliability Property

The reliability characteristics of a probability distribution are typically examined through its survival function, $S(x)$ and hazard rate function, $h(x)$, which are derived as follows:

$$S(x) = 1 - F(x) = \frac{e^{-x\beta}(2+2(1+x)\beta+x(2+x)\beta^2)}{2(1+\beta)}$$

$$h(x) = \frac{f(x)}{S(x)} = \frac{x(2+x)\beta^3}{2+2(1+x)\beta+x(2+x)\beta^2}$$

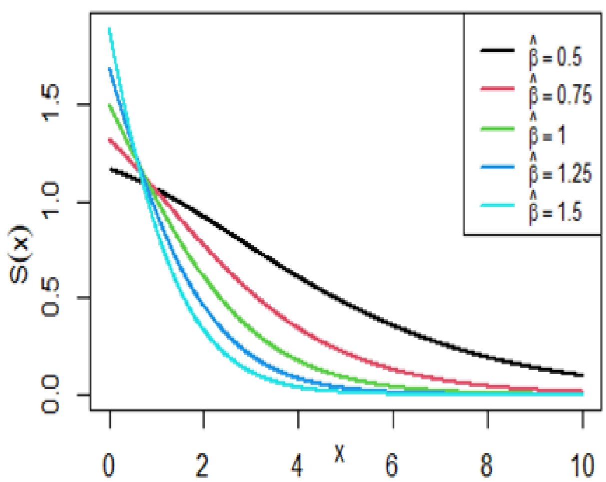

Figure 3: The Survival Function plots of the PGM Distribution for Different Values of $\hat{\beta}$

Figure 3 shows the survival function $S(x) = 1 - F(x)$, which represents the probability that the random variable exceeds a given value of x. As expected, $S(x)$ starts at 1 when $x = 0$ and declines toward 0 as x increases. The rate of decline depends on $\hat{\beta}$. Higher values of $\hat{\beta}$ cause the survival probability drops steeply, reflecting shorter expected lifetimes or faster decay. Lower values of $\hat{\beta}$ decline more gradually, indicating a higher likelihood of survival at larger x. This confirms the earlier findings: small β leads to longer tails and higher persistence, while large β corresponds to shorter tails and quicker decay.

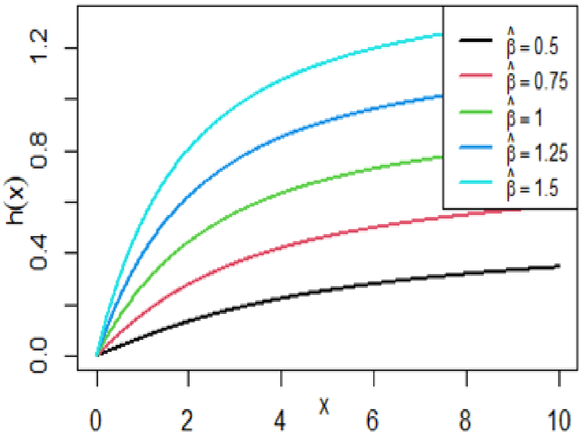

Figure 4: The Hazard Function plots of the PGM Distribution for Different Values of $\hat{\beta}$

1. Maximum Likelihood Estimation

The parameters of the PGM distribution can be estimated using the method of maximum likelihood, which provides desirable asymptotic properties including consistency, efficiency and normality. Let $X_1, X_2, \dots, X_n$ be a random sample of size n from the PGM distribution with probability density function given in Equation (1). The likelihood function is expressed as:

$$L(\beta|x_1,\dots,x_n) = \prod_{i=1}^{n} f(x_i;\beta)$$

$$= \prod_{i=1}^{n} \frac{\beta^3}{2(1+\beta)} x_i(x_i+2)e^{-\beta x_i}$$

$$= \left(\frac{\beta^3}{2(1+\beta)}\right)^n \prod_{i=1}^{n} x_i(x_i+2)e^{-\beta x_i} \qquad (10)$$

The corresponding log-likelihood function simplifies to:

$$\ell(\beta) = 3nln(\beta) - nln2 - nln(1+\beta) + \sum_{i=1}^{n} ln\, x_i + \sum_{i=1}^{n} ln\,(x_i+2) - \beta\sum_{i=1}^{n} x_i \qquad (11)$$

To obtain the maximum likelihood estimate (MLE) of β, we differentiate the log-likelihood function with respect to β and set the resulting expression to zero:

$$\frac{\partial\ell(\beta)}{\partial\beta} = \frac{3n}{\beta} - \frac{n}{1+\beta} - \sum_{i=1}^{n} x_i = 0$$

This yields the estimating equation:

$$\frac{3}{\beta} - \frac{1}{1+\beta} - \bar{x} = 0$$

Therefore,

$$\bar{x} = \frac{3+2\beta}{\beta(1+\beta)}$$

The maximum likelihood estimate $\hat{\beta}$ is obtained by solving this nonlinear equation numerically using iterative methods such as Newton-Raphson or Brent's optimization, as no closed-form solution exists. The existence and uniqueness of the MLE are guaranteed for $\bar{x} > 0$, which corresponds to the condition where the empirical mean exceeds the theoretical lower bound of the distribution's support.

The Fisher information matrix can be derived to construct asymptotic confidence intervals for the parameter β. The observed Fisher information is given by:

$$I(\hat{\beta}) = -\frac{\partial^2\ell(\beta)}{\partial\beta^2}\Big|_{\beta=\hat{\beta}} = \frac{3n}{\hat{\beta}^2} - \frac{n}{(1+\hat{\beta})^2}$$

For large samples, the MLE, $\hat{\beta}$, follows approximately a normal distribution with mean β and variance $I^{-1}(\hat{\beta})$, enabling the construction of $(1-\alpha)100\%$ confidence intervals as $\hat{\beta} \pm Z_{1-\alpha/2}\sqrt{I^{-1}(\hat{\beta})}$, where $Z_{1-\alpha/2}$ denotes the standard normal quantile.

RESULTS AND DISCUSSION

Simulation Study for MLE of the PGM Distribution

In this section we conducted a comprehensive simulation study to evaluate the finite-sample performance of the MLE procedure for the PGM distribution. The essence of this is to assess the consistency, unbiasedness and efficiency of the parameter estimates across varying sample sizes.

We employed Monte Carlo simulations with true parameter value $\beta = 0.5$ and sample sizes ranging from $n = 50 \dots 1000$ in increments of 50. For each sample size configuration, N = 1000 independent replications were performed to ensure robust statistical inference. Random samples were generated from the PGM distribution using the inverse transform method, which involved numerically solving the quantile function through root-finding algorithms.

The parameter estimation was carried out using maximum likelihood estimation via the L-BFGS-B optimization algorithm, which efficiently handles boundary constraints to ensure parameter positivity. The performance of the estimation procedure was evaluated using multiple metrics: bias, mean squared error (MSE) and empirical standard deviation.

Table 1: The results of the simulation study (for $\boldsymbol{\beta} = \mathbf{0.5}$)

n	$\hat{\beta}$	Bias	MSE	SD
50	0.4885	-0.0115	0.0016	0.0386
100	0.4839	-0.0161	0.0010	0.0273
150	0.4846	-0.0154	0.0007	0.0223
200	0.4843	-0.0157	0.0006	0.0191
250	0.4841	-0.0159	0.0005	0.0167
300	0.4826	-0.0174	0.0005	0.0150
350	0.4831	-0.0169	0.0005	0.0142
400	0.4833	-0.0167	0.0005	0.0137
450	0.4834	-0.0166	0.0004	0.0124
500	0.4833	-0.0167	0.0004	0.0120
550	0.4835	-0.0165	0.0004	0.0114
600	0.4834	-0.0166	0.0004	0.0106
650	0.4835	-0.0165	0.0004	0.0105
700	0.4841	-0.0159	0.0004	0.0101
750	0.4833	-0.0167	0.0004	0.0095
800	0.4828	-0.0172	0.0004	0.0094
850	0.4833	-0.0167	0.0004	0.0090
900	0.4830	-0.0170	0.0004	0.0089
950	0.4834	-0.0166	0.0003	0.0083
1000	0.4832	-0.0168	0.0003	0.0081

The simulation study results for the PGM distribution, in Table 1 and Figures 5, show that as the sample size n increases from 50 to 1000, the mean estimates of $\hat{\beta}$ remain close to the true value of 0.5, with a small negative bias that is approximately zero. The MSE and SD both declines steadily, reflecting improved estimator precision and efficiency sample size increases. These results confirm the asymptotic consistency of the estimators and improved estimation precision.

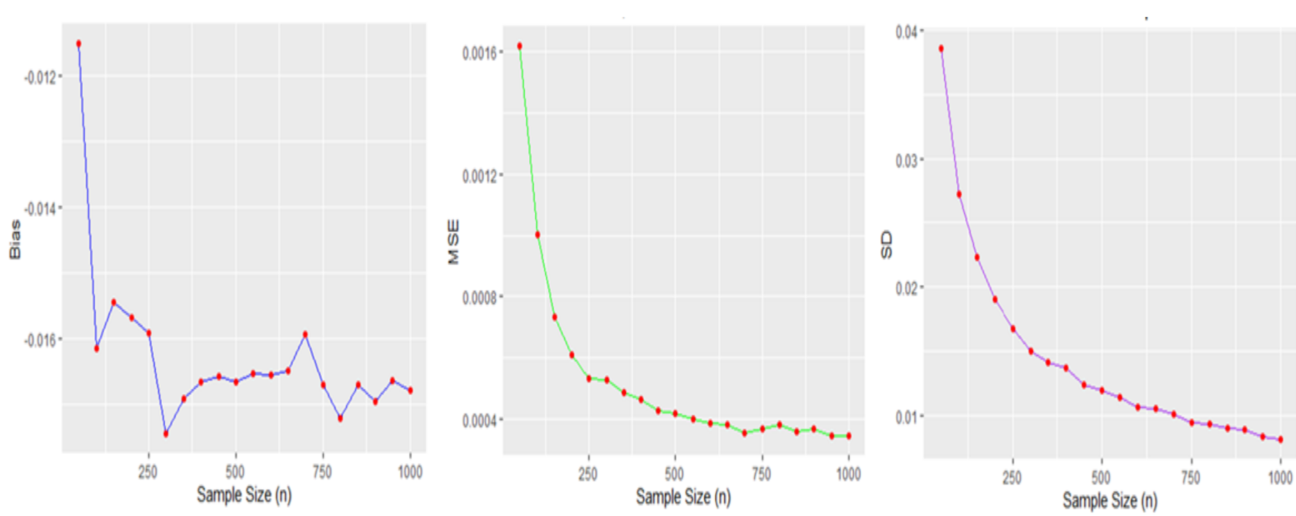

Figure 5: The Bias, MSE and SD plots of the simulation study

Application to Real Dataset

In this section we examine the practical performance of the proposed PGM distribution through empirical applications to three real-world datasets. The model's performance is evaluated against six well-established lifetime distributions to assess its comparative advantage in modelling reliability data.

Dataset Descriptions

The first datasets are about Fatigue life of 6061-T6 aluminum coupons (Birnbaum and Saunders, 1969), containing 101 observations of aluminum specimens oscillated at 18 cycles per second with maximum stress per cycle of 31,000 psi. The second data are on Aircraft window glass strength (Edwin et al., 1994), consisting of 31 measurements of glass strength from aircraft windows. This dataset characterizes material failure stresses and is particularly relevant for structural reliability analysis in aerospace applications. The third data are on the Glass fiber strength (Smith and Naylor, 1987), containing 63 measurements of 1.5 cm glass fiber strength obtained at the National Physical Laboratory, England. This dataset represents material strength properties crucial for composite material design and failure analysis.

Competing Models

The following established probability distributions considered as competing lifetime models (Eliwa et al., 2021):

1] **Exponentiated Half-Logistic (EHL)** with CDF:

$$F_{EHL}(x;\ \beta) = \left(\frac{1-e^{-x}}{1+e^{-x}}\right)^{\beta} \ ;\ x,\beta > 0$$

2] **Generalized Half-Logistic (GHL)** with CDF:

$$F_{GHL}(x;\ \beta) = 1-\left(\frac{2e^{-x}}{1+e^{-x}}\right)^{\beta} \ ;\ x,\beta > 0$$

3] **Lindley (Li)** with CDF:

$$F_{Li}(x;\ \beta) = 1-\left(1+\frac{\beta x}{1+\beta}\right)e^{-\beta x} ;\ x,\beta > 0$$

4] **Inverse Lindley (ILi)** with CDF:

$$F_{ILi}(x;\ \beta) = \left(1+\frac{\beta}{(1+\beta)x}\right)e^{-\beta x} ;\ x,\beta > 0$$

5] **Transmuted Half-Logistic (THL)** with CDF:

$$F_{TLi}(x;\ \beta) = \frac{(e^{x}-1)(1+2\beta+e^{x})}{(1+e^{x})^2} \ ;\ x > 0, |\beta\,| < 1$$

6] **Exponential (Exp)** with CDF:

$$F_{Exp}(x;\ \beta) = 1-e^{-\beta x} ;\ x > 0, \quad \beta > 0$$

Table 2: Results of MLEs and evaluation metrics for the datasets

	Distribution	Parameters ($\hat{\beta}$)	LogLik	AIC	AICC	BIC	HQIC
Dataset 1	**PGM**	**0.0433**	**-476.2584**	**954.5167**	**954.5575**	**957.1219**	**955.5711**
	Lindley	0.0289	-491.5548	985.1097	985.1505	987.7148	986.1640
	GHL	0.0148	-521.4222	1044.8444	1044.8853	1047.4496	1045.8988
	Exponential	0.0147	-522.4350	1046.8700	1046.9108	1049.4752	1047.9244
	InverseLindley	10.013	-627.0265	1256.0530	1256.0938	1258.6582	1257.1074
	EHL	100.01	-6304.515	12611.031	12611.072	12613.636	12612.086
Dataset 2	**PGM**	**0.0946**	**-121.0240**	**244.0479**	**244.1858**	**245.4819**	**244.5154**
	Lindley	0.0630	-126.9942	255.9884	256.1263	257.4224	256.4558
	GHL	0.0332	-136.5591	275.1182	275.2561	276.5522	275.5856
	Exponential	0.0325	-137.2644	276.5289	276.6668	277.9629	276.9963
	InverseLindley	10.0013	-152.0199	306.0398	306.1778	307.4738	306.5073
	EHL	100.0103	-790.9062	1583.8123	1583.9503	1585.2463	1584.2798
Dataset 3	**PGM**	**1.5841**	**-64.1453**	**130.2907**	**130.3563**	**132.4338**	**131.1336**
	EHL	2.0665	-64.2956	130.5912	130.6568	132.7344	131.4341
	GHL	0.9785	-77.4963	156.9927	157.0583	159.1359	157.8356
	Lindley	0.9961	-81.2784	164.5568	164.6224	166.7000	165.3997
	InverseLindley	1.8947	-85.8945	173.7891	173.8547	175.9323	174.6320
	Exponential	0.6636	-88.8303	179.6606	179.7262	181.8037	180.5035

Table 2 shows the performance results of the proposed distribution in comparism with the competing existing ones using three datasets. The PGM distribution indicates superior performance compared to the competing distributions. For Dataset 1, the PGM model achieves the lowest AIC value (954.52), which significantly outperforms the Lindley (985.11), GHL (1044.84), Exponential (1046.87), Inverse Lindley (1256.05) and EHL (12611.03) distributions. Similarly, in Dataset 2, the PGM distribution maintains its leading position with an AIC of 244.05, followed by Lindley (255.99), GHL (275.12), Exponential (276.53), Inverse Lindley (306.04) and EHL (1583.81). The results from Dataset 3 reveal a more competitive perspective, the PGM distribution (AIC = 130.29) slightly edging out the EHL distribution (AIC = 130.59), while both substantially outperform the remaining models. Across all the information criteria: AIC, AICC, BIC and HQIC, the PGM distribution consistently ranks first. This demonstrates its robust fitting capability and parameter parsimony.

CONCLUSION

In this study, the Parsimonious Gamma Mixture distribution is introduced as a robust, one-parameter model for lifetime data analysis. The proposed distribution consistently outperforms established competitors, including Lindley, Generalized Half-Logistic and Exponential models, across multiple materials science and engineering datasets, as evidenced by superior rankings in the metrics used. By integrating mixture-like flexibility within a single Gamma-based kernel, the PGM avoids over-parameterization while maintaining identifiability, closed-form statistical properties and interpretable tail behaviour governed by the parameter β. Maximum likelihood estimation of β demonstrates consistency and increasing precision with sample size via simulation study. The distribution is particularly well-suited for reliability applications, with future work recommended for extensions into regression frameworks, Bayesian implementations and censored data scenarios.

REFERENCE

Aderoju, S. (2021). Samade probability distribution: Its properties and application to real lifetime data. *Asian Journal of Probability and Statistics*, 14(1), 1–11. https://doi.org/10.9734/ajpas/2021/v14i130317.

Aderoju, S., & Adeniyi, I. (2022). On power generalized Akash distribution with properties and applications. *Journal of Statistical Modeling & Analytics (JOSMA)*, 4(1). https://doi.org/10.22452/josma.vol4no1.1

Aderoju, S. A., Dauda, K. A., & Olaifa, J. B. (2025a). Development of new flexible lifetime model: Its associated inferences and applications to cancer and COVID-19 data. *Journal of Science and Technology*. 30 (8). DOI: https://doi.org/10.20428/jst.v30i8.3024.

Aderoju, S. A., & Jolayemi, E. T. (2022). Power Hamza distribution and its applications to model survival time. *Journal of the Nigerian Statistical Association*, 34, 1–10.

Aderoju, S.A., Salau, G.M., Sanni, B.I., Adeshola, A.D., Jimoh, A.K., Wahab, T.O. and **Kalu, U.,** (2025b). The New Extended Exponential-Gamma (NEEG) Distribution: Properties and Applications to Infectious Disease Modelling. *Journal of Basics and Applied Sciences Research*, 3(4), 225-235. DOI: https://dx.doi.org/10.4314/jobasr.v3i4.26

Aleshinloye, N. I., Aderoju, S. A., Abiodun, A. A., & Taiwo, B. L. (2023). A new generalized gamma-Weibull distribution and its applications. Al-Bahir Journal for Engineering and Pure Sciences, 2(2), 90–100. https://doi.org/10.55810/2313-0083.1021

Birnbaum, Z. W., & Saunders, S. C. (1969). Estimation for a family of life distributions with applications to fatigue. Journal of Applied Probability, 6(2), 328–347.

Box, G. E. P., & Jenkins, G. M. (1976). Time series analysis, forecasting and control (Rev. ed.). Holden-Day.
Burnham, K. P., & Anderson, D. R. (2002). *Model selection and inference: A practical information-theoretic approach (2nd ed.)*. Springer.

Chrisogonus, K. O., George, A. O., & Samuel, U. E. (2020). Inverted power Rama distribution with application to lifetime data. *Asian Journal of Probability and Statistics*, 9(4), 1–21. https://doi.org/10.9734/ajpas/2020/v9i430231.

Eliwa, M. S., Altun, E., Alhussain, Z. A., Ahmed, E. A., Salah, M. M., Ahmed, H. H., et al. (2021). A new one-parameter lifetime distribution and its regression model with applications. *PLoS ONE*, 16(2), e0246969. https://doi.org/10.1371/journal.pone.0246969

Ferrari, S., & Cribari-Neto, F. (2004). Beta regression for modelling rates and proportions. *Journal of Applied Statistics*, 31(7), 799–815.

Frederick, A. O., Osuji, G. A., & Onyekwere, C. K. (2022). Inverted power Ishita distribution and its application to lifetime data. *Asian Journal of Probability and Statistics*, 18(1), 1–18. *https://doi.org/10.9734/ajpas/2022/v18i130433*

Fuller, E. R., Freiman, S. W., Quinn, J. B., Quinn, G. D., & Carter, W. C. (1994). Fracture mechanics approach to the design of glass aircraft windows: A case study. *In Window and dome technologies and materials IV*. Vol. 2286, pp. 419–430). *International Society for Optics and Photonics.*

Ghitany, M. E., Atieh, B., & Nadarajah, S. (2008). Lindley distribution and its application. *Mathematics and Computers in Simulation*, 78(4), 493–506. https://doi.org/10.1016/j.matcom.2007.06.007

Johnson, N. L., Kotz, S., & Balakrishnan, N. (1995). *Continuous univariate distributions (2nd ed., Vol. 2)*. John Wiley & Sons.

Lindley, D. V. (1958). Fiducial distributions and Bayes' theorem. *Journal of the Royal Statistical Society: Series B (Methodological)*, 20(1), 102–107.

Marshall, A.W., Olkin, I. (2007). *The Exponential Distribution*. In: Life Distributions. Springer Series in Statistics. Springer, New York, NY. https://doi.org/10.1007/978-0-387-68477-2_8

Mazucheli, J., Menezes, A. F. B., & Chakraborty, S. (2018). On the one-parameter unit-Lindley distribution and its associated regression model for proportion data. *Journal of Applied Statistics*, 46(4), 700–714. https://doi.org/10.1080/02664763.2018.1511774

Frank, Okpala I., Happiness O. Obiora-Ilouno, and Omoruyi A. Frederick. 2023. "Inverse Hamza Distribution: Properties and Applications to Lifetime Data". *Asian Journal of Probability and Statistics* 23 (1):46-64. https://doi.org/10.9734/ajpas/2023/v23i1496.

Omoruyi, F., Onyekwere, C., Udofia, E., Ejiofor, F., & Victor, C. (2023). Inverse power Hamza distribution: Properties and applications to lifetime data. *Journal of Xidian University*, 17, 2023. https://doi.org/10.37896/jxu17.7/074

Omoruyi, F. A., Omeje, I. L., Anabike, I. C., & Obulezi, O. J. (2023). A new variant of Rama distribution with simulation study and application to blood cancer data. *European Journal of Theoretical and Applied Sciences*, 1(4), 389–409. https://doi.org/10.59324/ejtas.

Shanker R, Fesshaye H, Selvaraj S. (2015). On modeling of lifetimes data using exponential and lindley distributions. *Biom Biostat Int J.*; 2(5):140-147. DOI: 10.15406/bbij.2015.02.00042

Shanker, R., & Shukla, K. K. (2017). Ishita distribution and its applications. *Biometrics & Biostatistics International Journal*, 5(2), 39–46. https://doi.org/10.15406/bbij.2017.05.00126

Shukla, K. K. (2018). Pranav distribution with properties and its applications. *Biometrics & Biostatistics International Journal,* 7(3), 244–254. https://doi.org/10.15406/bbij.2018.07.00215

Shukla, K. K. (2019). A comparative study of one-parameter lifetime distributions. *Biometrics & Biostatistics International Journal*, 8(4), 111–123. https://doi.org/10.15406/bbij.2019.08.0028

Shukla, K. K. (2021). Inverse Ishita distribution: Properties and applications. *Reliability: Theory & Applications*, 1(61), 16.

Smith, R. L., & Naylor, J. C. (1987). A comparison of maximum likelihood and Bayesian estimators for the three-parameter Weibull distribution. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 36(3), 358–369.