



Modification of Wilks' Lambda Test Statistic in Multivariate Analysis of Variance Test



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ABSTRACT

This study modified Wilks' Lambda Test Statistic in Multivariate Analysis of Variance Test. Simulation was done using R. The sample sizes utilized were 5, 10, 15, 20, 25, 50 and 100; Number of variables and Number of groups $p = g = 2, 3$ and 4 with equal, unequal samples sizes and variance co – variance matrix. The comparison were done at one level of $\alpha = 0.05$ using power of the test and type I error rate. From the result obtained under type I error rate, it was indicated that when $g = 2, p = 2, p = 3, p = 4, g = 3, p = 2, p = 3, p = 4, g = 4, p = 2, p = 3$, when $g = 4, p = 4$ and compares with chi – square value. It was noticed that the proposed test statistic performs better at all sample sizes and across all variables when the sample sizes were large (i.e 50 and 100) but not perform better when the sample sizes are small. Under power of the test, it was observed that when $g = 2$ and $p = 2, p = 3$ and $p = 4, g = 3$ and $p = 2, p = 3, p = 4, g = 4, p = 2, p = 3, p = 4$ and compares all with chi – square value. It was noticed that the proposed test statistic does not performs better at all sample sizes and across all variables.

Keywords:

Modified Wilks' Lambda;
Type I error rate;
Power of the test, Manova.

INTRODUCTION

Over the years, many researchers compared the the performance of the four multivariate analysis of variance test statistic (MANOVA) when the sample size is high and small, when the mean vectors are different, and when the mean vectors are equal. Groups are compared on a collection of dependent variables at the same time using multivariate analysis of variance (MANOVA). The MANOVA approach makes a single compariason rather than testing group differences with many independent ANOVAs and running the risk of increasing family – wise error (chance of one or more Type I errors). Only when there is a relationship between the several dependent variables and the expected pattern of group differences for each dependent variable points in the same direction is the MANOVA appropriate (Winter, 2025). Individual items, multiple scale scores, or other related measures might be used as the multiple measures. The ability of various psychotherapy approaches (independent variables) to lessen psychological distress could be an example, where a number of psychological distress measures, such as depression, anxiety, and perceived stress, are examined collectively (as dependent variables).

As an alternative, one might examine a number of depression subscales, including somatic symptoms, positive and negative effects, and others. The convenience of a different kind of omnibus test of all the measures at once is offered by MANOVA.

The majority of researchers across a wide range of specializations gather data sets that are multivariate in nature. It may make sense in certain situations to look at each variable independently, but in the majority of circumstances, all the variables must be looked at once in order to completely undrestand the structure and important aspects of the data. A collection of measurements or observations performed on various topics, patients, objects, or other entities of interest make up the data that we are mainly interested in. A measure of the overall probability of selecting two or more random vectors or means is provided by the MANOVA test statistic (Chatfield and Collins, 1995). This method uses a set of factors functioning as independent variables to evaluate group differences across several dependent variables at once. MANOVA is a statistical method that examiners the connection between two or more dependent variables and a number of independent variables, often known as treatments (Tabachnick and Fidel, 2012).

Consequently, it is a continuation of univariate analysis of variance. After the experiment, the effect of any uncontrolled independent factors, or covariates on the dependent variables is eliminated using multivariate analysis of covariance in conjunction with MANOVA. Covariates are the name given to these uncontrollable independent variables. Measurement of several variables is a common task for researchers in the biological, physical, and social sciences (Richard and Michael, 1987). Multivariate analysis is intrinsically challenging since it requires an understanding of the relationships between numerous response variables, and the sheer volume of the data may definitely overwhelm the human mind. Specifically, compared to univariate contexts, certain matrix algebraic principles are needed for the various multivariate statistical procedures.

It is crucial to compare the new modified test statistic's performance to that of the Wilks' Lambda, in order to determine whether the modified test statistic performs better than the original Wilks' Lambda test statistic. This is meant to be done in order to provide an alternate method for the original Wilks Lambda when the sample size is bigger. The aim of this study is to propose a new modified Wilks' Lambda test statistic and its objective is to compare the performance of modified Wilks' Lambda test statistic with the existing Wilks' Lambda and evaluate how well the new test statistic performs in comparison to the current process when sample size varies and variance – covariance matrices are known and unknown.

(Hend, 2020), In his dissertation, to more effectively smooth raw data, a revolutionary multivariate nonparametric techniques was put forth, which narrowed the solution's dimension to a few intriguing parameters. The suggested strategy made use of techniques that took use of kernel density estimated (KDE), which may be used to test hypotheses regarding the primary effects and interaction effect in the two – way layout as well as the equality of location parameters in the one – way layout. In order to do multivariate analysis of variance (MANOVA) and test hypotheses against different group means in a variety of contexts, including one, multivariate kernel – based tests were first created. After that, the asymptotic distributions and asymptotic properties of the suggested approaches were examined. The small – sample behaviour of the suggested nonparametric kernel – based test statistics for the one – and two – way layout was then examined using simulations. The suggested nonparametric kernel – based techniques were then contrasted with their conventional parametric counterparts for one – way and two – way layout. Lastly, an actual picture dataset was used to test the suggested nonparametric kernel – based techniques. The findings of this dissertation demonstrated that when the underlying distribution of the data is non – normal, the suggested nonparametric kernel – based methods have power than the corresponding parametric methods, such as

MANOVA, in identifying group differences in multivariate settings.

(Sheu, 2020), evaluates, using power analysis, the effectiveness of multivariate treatment tests (Wilk's Lambda, Hotelling Lawley, Roy's biggest root, and Pillai) on multivariate Sudoku square design, models. The power of these four tests for the four multivariate Sudoku square design models was compared using Monte Carlo simulation. In his research, the power difference between two tests with the same sample size was measured as an interval value of ± 0.062 . If the difference between the test's powers is ± 0.062 , the test is deemed powerful or advantageous. According to the power test results, Hotelling – Lawley outperforms the other three tests at $p = 2$, but at $p = 3$, in all multivariate Sudoku models, Wilks' Lambda test has a power advantage over other tests.

(Iftikhar, 2020) uses simulated data from agricultural trials to demonstrate MANOVA. On simulated data, the proper design techniques for analysis, interpretation, and conclusion are applied. For example, in a completely randomized design (CRD), three elements are taken into account in a factorial layout. All possible combinations of the two levels of each irrigation, variety, and nitrogen source are taken into consideration in order to quantify the yield and plant height, two linearly connected responses. Two parameters, let's say plant height and yield, are simulated using data with different correlation coefficient magnitudes (low, moderate and high) between the two response variables. The comparison of MANOVA and ANOVA shows that even a slight correlation between dependent variables significantly impacts the main effects and interactions of the three independent variables. When the linear relationship between traits is low to moderate, ANOVA and MANOVA yield contrasting results for some main effects and interactions. Moreover, with highly correlated traits, MANOVA drastically changes the interpretation of effects compared to separate ANOVAs for each trait.

(Béatrice, 2022) emphasizes the construction and examination of residuals in the so called GMANOVA – MANOVA model. The Extended Growth Curve Model is the model's special case. It consists of two terms, one of which models the growth curves or profiles and the other of which represents the relevant covariables. In disciplines including biology, medicine, epidemiology, and economics, this model is helpful for examining growth curves in brief time series. Additionally, residuals have been thoroughly examined and utilized in the literature to assess the adequacy of univariate linear models. The maximum likelihood estimators of the model's parameters are used in this thesis to build a fresh pair of residuals. An individual's distance from the group means is shown by one residual, and assumptions on the mean structure are verified by a second residual. These residuals' various attributes are checked, and their meaning is examined. Additionally, the empirical distributions of the extreme

elements in the residuals are obtained using parametric bootstrap. Lastly, the MANOVA model's testing of bilinear limitation is examined. It is possible to demonstrate that the MANOVA model with bilinear constraints is only a GMANOVAMANOVA model. Additionally, it can be demonstrated that the likelihood ratio test is provided as a function of the GMANOVA – MANOVA model's residuals, which may be used to assess the model's suitability and test the bilinear hypothesis.

(Sarah, 2017) a contemporary and reliable analysis of longitudinal and multivariate data in factorial experiments, non – parametric statistical inference techniques are crucial. Although statistical software packages implement existing approaches that rely on particular distributional assumptions of the data (multivariate normality and/ or equal covariance matrices), user – friendly software that can be used for the analysis of the data that do not fulfill the aforementioned assumptions and provide accurate estimates of the p value and confidence interval is needed. Thus, the publicly accessible R package MANOVA. RM incorporates recently created nonparametric statistical techniques based on bootstrap and permutation procedures, which do not need multivariate normality or particular covariance matrices. A graphical user interface is included in the software for believable academic and other educational uses. The methods' use is demonstrated by a number of inspiring cases.

(Okoli, 2022). Lawley – Hotelling, Pillai's trace, Roy's biggest root, and Wilks' lambda are the four test statistics used in MANOVA that were compared for power. The data used to compare the four test statistics under the Multivariate Gamma and Multivariate Normal distribution was simulated using R Statistics. Sample sizes of 10, 20, 30, 40, 100, 200, 300, 400, 600, 700, 800 and 1000 were employed; there were also equal and unequal samples for variables ($p = 2, 3$ and 4) and a variance – covariance matrix. Using test power, the comparison was conducted at two levels of significance ($\alpha = 0.01$ and 0.05). Because it has the highest powers, the results showed that the Roy's largest Root test statistic is superior to all other test statistic taken into consideration when $p = g = 2$. The analysis's outcome also demonstrated that, for both small and large sample sizes, Wilks' lambda outperformed all other test statistics for $p = g = 3$ and $p = g = 4$. The results equally showed that when the data are multivariate normal and Gamma with $g = 2$ and $p = 2$ the power of the four test statistics from best to least is Roy's largest root, followed Lawley's trace = Pillai's trace and the least is Wilks' Lambda at significant levels of 0.01 and 0.05 for equal and unequal samples. The power of the four statistics, arranged

from best to least, is Wilks' Lambda, Pillai's trace = Lawley's trace, and Roy's greatest root, respectively, when the data are multivariate normal and gamma, and $p = g = 3$ and $p = g = 4$. The results clearly show that Wilks' Lambda should be used when $p = g = 3$ and $p = g = 4$, and Roy's biggest root should be applied when $p = g = 2$. In order to get definitive results as efficiently as possible, this study will assist researchers in designing investigations with regulated odds of discovering a relevant effect.

(Abdullahi, 2020). The two – way multivariate analysis of variance (MANOVA), which is extremely sensitive to the effects of outliers, is mostly employed to test hypotheses using the traditional Wilks' statistic. Many academics have looked at alternative options due to the non – robustness of test statistics based on normal theory. In this work, we introduced a robust version of the Wilks' test statistic that is based on reweighted minimum covariance determinant estimates (RMCD), which are extremely efficient and resilient. The performance of the test statistics under different distributions is assessed using Monte Carlo simulation. Furthermore, test power and type I error rate data are regarded as statistical tools for comparing test statistics.

MATERIALS AND METHODS

Although MANOVA can be used with any number of groups (i.e., levels of any independent variable), k , and any number of dependent variables, p , the Hotelling's T^2 is expressed equation¹ in terms of two groups. Typically, MANOVA software processes (such as R – package) generate a number of closely similar tests. When the result is significant, it means that there are differences between the groups on the dependent variables taken together. Wilks' lambda is the most basic and direct in terms of its analogous relation to F in univariate ANOVA, Mirroring the ratio of mean square between to mean square in the equation. Equation² below is displayed using E and H , which are matrices of the sums of square of y and their cross – products (the variance and covariance matrices without dividing by df)

$$T^2 = \frac{n_1 n_2}{n_1 + n_2} (\bar{y}_1 - \bar{y}_2)' S^{-1} (\bar{y}_1 - \bar{y}_2) \quad (1)$$

Wilks Lambda

$$\Lambda = \prod_{i=1}^s \frac{1}{1 + \lambda_i} = \frac{|E|}{|E + H|} \quad (2)$$

Equ² above can also be expressed as the function of eigen values $\lambda_1, \lambda_2, \dots, \lambda_i$ of $(E^{-1}H)$ where $\Lambda = tr(E^{-1}H)$ and for which Λ range for 0 to 1 i.e $0 \leq \Lambda \leq 1$.

Where

$$E = \sum_{i=1}^t \sum_{j=1}^b (x_{ij} - \bar{x}_{i.})(x_{ij} - \bar{x}_{i.})^1 \quad (3)$$

$$H = \sum_{i=1}^t n_i (\bar{x}_{i.} - \bar{x}_{..})(\bar{x}_{i.} - \bar{x}_{..})^1 \quad (4)$$

$$\text{Power} = P(\text{Reject } H_0 | H_1 \text{ is true}) = P(Q > \chi^2_{(1-\alpha)} df | Q \sim \chi^2_{df} \Lambda^*)$$

Equivalently, since the **Type II error probability** (β) is the probability of failing to reject H_0 when H_1 is true, the power is

$$\text{Power} = 1 - \beta$$

$$\text{Power} = 1 - F_{\chi^2_{df}}(\Lambda^*)(\chi^2_{(1-\alpha)} df) \quad (5)$$

The existing Wilks' lambda $\Lambda = \prod_{i=1}^s \frac{1}{1+\lambda_i} = \frac{|E|}{|E+H|}$ is modified to

$$\Lambda^* = \sum_{i=1}^s \frac{1}{\lambda_i} = \text{tr}[(E^{-1}H)]^{-1}$$

for $\Lambda^* \sim \chi^2_{n(p-1)(g-1)}$ for which n_i represents total sample size, g = group size and p = population size and when $i \geq 1$.

The modified Wilks' lambda Λ^* was iterated in k^{th} times in which $k = 1000$. The k^{th} iterated result was checked with the chi – square to know how many time will test statistic greater than chi – square value at varying sample sizes of 5, 10, 15, 20, 25, 50 and 100 for group sizes (g) are 2, 3 and 4 and number of variables (p) are also 2, 3 and 4. At a various level of groups, observing which of them

perform better under type I error rate, power of the test for both null hypothesis and alternative hypothesis using varying sample size.

Data generation processes

The data used for this study was simulated and analyzed using R statistical package. The n covariates were simulated $X_1, X_2, \dots, X_n$ from the multivariate normal distribution with mean vector and variance – covariance matrix vector as shown below

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdot & \cdot & \cdot & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdot & \cdot & \cdot & \sigma_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdot & \cdot & \cdot & \sigma_{nn} \end{pmatrix}$$

With varying sample size $n = 5, 10, 15, 20, 25, 50$, and 100 , the variance covariance matrix of the covariates is obtained for the various values of n and α is fixed as 95%

```
## When p = 4, group = 4
test44 <- function(k,m,n) {
  library(MASS)
  m1=c(4,8,12,18)
  m2= m1*m
  m3= m2*m
  m4= m3*m
  s=matrix(c(2,1,1,1,1,4,2,1,1,2,6,1,1,1,1,8),ncol=4)
  for (i in 1:k) {
    x1=mvnrnorm(n,m1,s)
    x2=mvnrnorm(n,m2,s)
    x3=mvnrnorm(n,m3,s)
    x4=mvnrnorm(n,m4,s)
    x=rbind(x1,x2,x3,x4)
    factor=sort(rep(1:4,n))
    fit = manova(x~factor)
    ss <- summary.manova(fit)$SS
    e <- ss$Residuals
    h <- ss$factor
    w1 <- solve(e)%*%h
    w11 <- eigen(w1)$values
    ww1 <- as.numeric(sum(w11))
    output <- c(output, ww1)
  }
  return(output)
}

return(output)
test <- function(k,n) {
  library(MASS)
  m1=c(2,3,4)
  m2=c(2,3,4)
  m3=c(2,3,4)
  s=matrix(c(2,1,1,1,4,2,2,2,8),ncol=3)
  for (i in 1:k) {
    x1=mvnrnorm(n,m1,s)
    x2=mvnrnorm(n,m2,s)
    x3=mvnrnorm(n,m3,s)
    x=rbind(x1,x2,x3)
    factor=sort(rep(1:3,n))
    fit = manova(x~factor)
    ss <- summary.manova(fit)$SS
    e <- ss$Residuals
    h <- ss$factor
    w <- solve(e)%*%h
    w2 <- eigen(w)$values
    ww2<- as.numeric(1/sum(w2))
    output <- c(output,ww2)
  }
}
```

RESULTS AND DISCUSSION

TABLE 1: Type I error rate $\alpha = 0.05$ when $g = 2$

Number variable (p)	Sample size (n)	Proposed test
2 Chi – square = 5.991465	5	0.003
	10	0.000
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
3 Chi – square = 7.814728	5	0.066
	10	0.002
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
4 Chi – square = 9.487729	5	0.274
	10	0.020
	15	0.001
	20	0.000
	25	0.000
	50	0.000
	100	0.000

Table 1 shows the result obtained when $g = 2$ and $p = 2$, $p = 3$ and $p = 4$. Comparing chi – square value obtained (5.991465, 7.814728 and 9.487729) to all the group size, variables and sample sizes. It was noticed that the proposed test statistic performs better at when the group size is 2, number of variable is 2 and at larger sample sizes.

Table 2: Type I Error Rate when $\alpha = 0.05$ when $g = 3$

Number variable (p)	Sample size (n)	Proposed test
2 Chi – square = 5.991465	5	0.000
	10	0.000
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
	100	0.000
3 Chi – square = 7.814728	5	0.036
	10	0.001
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
	100	0.000
	5	0.214
	10	0.029

4 Chi – square = 9.487729	15	0.004
	20	0.001
	25	0.000
	50	0.000
	100	0.000

Table 2 shows the result obtained when $g = 3$ and $p = 2$, $p = 3$ and $p = 4$. Comparing chi – square value obtained (5.991465, 7.814728 and 9.487729) to all the group size, variables and sample sizes. It was noticed that the proposed test statistic performs better at when the group size is 3, number of variables is 2 for all sample sizes. But, when group size is 3, number of variables is 3, it was noticed that the proposed test statistic performs better when the sample sizes are larger and it was also noticed that the proposed test statistic performs better when the sample sizes are larger at group size is 3 and number of variables is 4.

Table 3: Type I Error Rate when $\alpha = 0.05$ when $g = 4$

Number variable (p)	Sample size (n)	Proposed test
2 Chi – square = 5.991465	5	0.000
	10	0.000
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
3 Chi – square = 9.487729	5	0.013
	10	0.000
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
4 Chi – square = 9.487729	5	0.119
	10	0.007
	15	0.001
	20	0.000
	25	0.000
	50	0.000
	100	0.000

Table 3 above shows the result obtained when $g = 4$ and $p = 2$, when $g = 4$ and $p = 3$ and when $g = 4$, $p = 4$. Comparing chi – square value obtained (5.991465, 7.814728 and 9.487729) to all the group size, variables and sample sizes. It was noticed that the proposed test statistic performs better at when the group size is 4, number of variables is 2 for all sample sizes. But, when group size is 4, number of variable is 3, it was noticed that the proposed test statistic performs better when the sample sizes are larger value i.e from 20, 25, 50 and 100 and it

was also noticed that the proposed test statistic performs better when the sample sizes are larger at group size is 4 and number of variable is 4, the proposed test statistic perform better at sample sizes are 20, 25, 50 and 100.

Table 4: Power of the test when $\alpha = 0.05$ for $g = 2$

Number variable (p)	Sample size (n)	Proposed test
2 Chi – square = 5.991465	5	0.036
	10	0.002
	15	0.000
	20	0.000
	25	0.000
	50	0.000
	100	0.000
3 Chi – square = 5.991465	5	0.778
	10	0.601
	15	0.496
	20	0.452
	25	0.388
	50	0.239
	100	0.102
4 Chi – square = 5.991465	5	0.989
	10	0.988
	15	0.990
	20	0.986
	25	0.998
	50	0.999
	100	1.000

Table 4 shows the result obtained when $g = 2$ and $p = 2$, $p = 3$ and $p = 4$. Comparing chi – square value obtained (5.991465, 7.8147729 and 9.487729) to all the group size, variables and sample sizes. It was noticed that there is no evidence that the proposed test statistic does not performs better when the group size is 2, number of variables is 2 and at a larger sample sizes, but when the group size is 3, number of variable is 3, it was noticed that there is evidence that the proposed test statistic does not performs better at all level of sample sizes and also there is evidence that the proposed test statistic does not perform better when the sample sizes at both smaller and larger sample sizes.

Table 5: Power of the test when $\alpha = 0.05$ for $g = 3$

Number variable (p)	Sample size (n)	Proposed test
2	5	0.411
	10	0.204
	15	0.108
	20	0.107
	25	0.030
	50	0.001

Chi – square = 5.991465	100	1.000
3 Chi – square = 7.8147729	5	0.999
	10	1.000
	15	1.000
	20	1.000
	25	1.000
	50	1.000
	100	1.000
4 Chi – square = 9.487729	5	1.000
	10	1.000
	15	1.000
	20	1.000
	25	1.000
	50	1.000
	100	1.000

Table 5 shows the result obtained when $g = 3$ and $p = 2$, $p = 3$ and $p = 4$. Comparing chi – square value obtained (5.991465, 7.8147729 and 9.487729) to all the group size, variables and sample sizes. It was noticed that there is evidence that the proposed test statistic does not performs better at all sample sizes except at the sample size is = 50. When the group size is 3, number of variables is 3 and at all sample sizes, the proposed test statistic performs considerably better. But when the group size is 3, number of variables is 4, it was noticed that there is evidence that the proposed test statistic does not performs better at all sample sizes.

Table 6: Power of the test when $\alpha = 0.05$ for $g = 4$

Number variable (p)	Sample size (n)	Proposed test
2 Chi – square = 5.991465	5	0.905
	10	0.931
	15	0.950
	20	0.971
	25	0.974
	50	0.997
	100	1.000
3 Chi – square = 7.8147729	5	1.000
	10	1.000
	15	1.000
	20	1.000
	25	1.000
	50	1.000
	100	1.000
4	5	0.999
	10	1.000
	15	1.000
	20	1.000
	25	1.000
	50	1.000

Chi – square = 9.487729	100	1.000
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Table 6 shows the result obtained when $g = 4$ and $p = 2$, $p = 3$ and $p = 4$. Comparing chi – square value obtained (5.991465, 7.8147729 and 9.487729) to all the group size, variables and sample sizes. It was noticed that there is evidence that the proposed test statistic does not performs better at all sample sizes and across all variables.

The results obtained in Table 1, $g=2$ and $p=2$, $p=3$ and $p=4$; Table 2, $g=3$ and $p=2$, $p=3$ and $p=4$; and Table 3, $g=4$ and $p=2$, and when $g=4$ and $p=3$, show that the proposed test statistic performs better when the group size is 2, 3, or 4 and the number of variables is 2 and at larger sample sizes. This goes alongside the result obtained by Okoli (2022) that Wilks' Lambda should be used when $p=g=3$ and $p=g=4$, and Roy's biggest root should be applied when $p=g=2$ at a larger sample size in order to get definitive results as efficiently as possible. Also, the results obtained in Table 4, Table 5 and Table 6 when $g=2$ and $p=2$, $p=3$ and $p=4$, $g=3$ and $p=2$, $p=3$ and $p=4$ and $g=4$ and $p=2$, $p=3$ and $p=4$ go alongside the result obtained by Sheu (2020), who evaluates, using power analysis, the effectiveness of multivariate treatment tests (Wilk's Lambda, Hotelling Lawley, Roy's biggest root, and Pillai). The power of these four tests were compared and. According to the power test results, Hotelling-Lawley outperforms the other three tests at $p = 2$, but at $p = 3$, in all multivariate Sudoku models, Wilks' Lambda test has a power advantage over other tests.

CONCLUSION

The new modified wilks' Lambda λ^* test statistic has been studied. The new test statistic was used to carried out analysis for a simulated data under type I error rate and power of the test for varying sample sizes. Based on the result obtained from the table and the analysis, it was concluded that the proposed test statistic (modified wilks' Lambda λ^*) perform better under both type I error rate and power of the test when the sample sizes are larger and at all number of groups and all number of variables.

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