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A Set Covering Model for Optimizing Selection of Contract Bidding in a Strategic Sourcing Process: A Case Study of the Nigerian Airforce

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ABSTRACT
The sourcing decisions of contract bidding in a Military organization is complex to manage largely due to the increasing complexity of products requirements, multiple suppliers, and the nature of Air Force Hardware and Software installations pricing structures. This paper presents a set-covering model (formulation) that allows the user to select the most economical bid among offerors (contractors) that meet all the critical product requirements while minimizing the total cost. The optimization process is carried out in two phases. The first phase deals with the construction of a biddable combination matrix by mapping out the critical product requirements against the offerors' (contractors) specifications. In the second phase, the model makes an optimal assignment of offerors to each feasible or contracting product by utilizing economies of scales offered by credible offerors volumes. This gives an optimization model for selecting the set of bid among multiple offerors' proposals for installation services. The selection achieves the most favorable objective based on balancing the confidence performance level in past performance of the offerors and the cost to the Air Force. The research findings based on a realistic scenario demonstrate improvements in both overall performance and cost than the status quo.

Keywords:
Optimization,
Set Covering Problem,
Strategic Sourcing,
Procurement,
Offerors.

INTRODUCTION

Today's government agencies are operating in an environment characterized by countless economic and political disruptions to their sources of supplies and services. The Nigeria Air Force (NAF) annually procures billions of Naira worth of systems, supplies and services in support of the national military strategy. The NAF Fiscal Year 2024 procurement budget includes N209.78 billion for defense-related supplies and services (Government Accountability Office, 2009). Faced with these fiscal battles of budget cuts and resource constraints, the NAF must monitor its procurement process to ensure a continuous flow of critical supplies and services. The NAF procurement process will continue to increase in importance as the NAF acquires mission-critical and complex supplies and services. The NAF has been undergoing a transformation in terms of how it manages its procurement function to include its personnel, processes, practices and policies.

The NAF's procurement function is transforming from a transaction-oriented perspective to a strategic-oriented enterprise. No longer viewed as a tactical, clerical or administrative function, the procurement function is gaining enhanced status and importance as leading organizations including the NAF understand and realize procurement's importance in achieving organizational strategic objectives as well as procurement's impact on competitive advantage.

Furthermore, organizations are including procurement objectives in the development of corporate strategy and have placed great emphasis on developing corporate procurement strategies. One aspect of this transformation is the use of a strategic sourcing approach for the procurement of installation-level services. The Air Force has taken the lead in adopting a strategic sourcing approach for the procurement of its major installation-level services.

Adopting the NAF's strategic sourcing process in our context, this research discusses the development and application of an optimization using set convex problem formulation for evaluating and selecting an offerors' proposal in an optimal bidding, sourceselection approach. The objective of the research is to show how a pricing optimization (PO) model can be successfully used in optimal bidding approaches, in which multiple offerors propose at multiple locations. Specifically, this research applies combinatorial optimization to find a set of bids that will achieve the most favorable objective. This objective is based on balancing the confidence level in past performance of the offerors and the cost to the Air Force. This research is an innovative application of operations research to NAF contract management.

Introduction to Strategic Sourcing and Contract Bidding

Strategic sourcing involves a systematic and data-driven approach to procurement, aiming to optimize purchasing decisions by evaluating cost, quality, and supplier capabilities. In multi-sourcing environments, where organizations receive bids from multiple vendors for different service or product bundles, selecting the optimal set of suppliers is crucial.The selection process often involves solving combinatorial problems where constraints like coverage, cost, and capacity must be addressed simultaneously. In this context, mathematical programming models—particularly Set Covering Problems (SCP)—have become popular for modeling supplier selection and contract bidding optimization.

The Set Covering Problem (SCP) Framework

The SCP is a classical optimization model that seeks the smallest (or least-cost) subset of options that collectively cover all required elements. In the context of contract bidding: **Elements** represent sourcing requirements (products, regions, or service types), **Sets** represent bids or contracts offered by suppliers. The objective is to select the minimum-cost combination of bids that covers all sourcing requirements, Chvátal (1979). One of the earliest mathematical formulations of SCP, providing the foundation for later heuristic and approximation approaches. See Muyi *et al* (2024).
Caprara *et al.* (2000), Introduced enhanced algorithms and polyhedral approaches for large-scale SCPs.

Application in Strategic Sourcing

Several studies adapt SCP to model strategic sourcing problems, integrating constraints such as supplier capacities, quality thresholds, and bundling effects.Relevant Studies **s**uch as Wang *et al.* (2009) proposed a mixed-integer linear programming (MILP) model for sourcing bid evaluation that closely resembles set covering, accounting for bundled offers. Aissaoui *et al*. (2007) provided a comprehensive review of e-procurement and sourcing decision models, highlighting set covering as a preferred technique for multi-criteria contract award problems. De Boer *et al* (2001) discussed supplier selection decision methods and indicated the potential of SCP when dealing with discrete bid sets.

Variations and Enhancements of SCP in Bidding Context

Real-world contract bidding problems often require extending the basic SCP to accommodate additional features like Budget constraints (Set Covering with Budget Constraints), Risk mitigation (Robust or Stochastic SCP), Multi-objective optimization (e.g., cost vs. sustainability), Capacitated SCP (CSC): Useful when suppliers have limits on how much they can supply.Examples include Özdemir and Yavuz (2015) integrated supplier risk into a capacitated SCP for procurement decisions and Che and Wang (2012) applied a fuzzy multi-objective SCP model to manage trade-offs between cost, delivery time, and quality in sourcing.

Solution Approaches

Due to NP-hardness, exact algorithms become impractical for large datasets. As a result, heuristics and metaheuristics are widely used exact methods: Branch-and-bound, branch-and-cut (for small instances).Heuristics such as Greedy algorithms and Lagrangian relaxation. Metaheuristics such as Genetic Algorithms, Simulated Annealing, Tabu Search. Illustrative Examples include among others, Beasley and Chu (1996) where they developed a Genetic algorithm for SCP aanddemonstrated its effectiveness in large-scale bid optimization.Yaghini et al. (2011) also developed a hybrid method combining Lagrangian relaxation and genetic algorithms for solving large sourcing SCPs.

Gaps and Future Research Directions

Among the gaps one can identify *dynamic sourcing* where many models assume static bid sets; few address evolving supplier availability or rolling contracts. Sustainability and ESG metrics: Limited integration of environmental or social criteria in SCP for sourcing.Real-time decision-making where new frameworks are needed to integrate real-time data (e.g., supplier performance, market volatility). Digital platforms and AI integration where there is growing potential to combine SCP models with AI-driven analytics for more adaptive sourcing strategies.
From the foregoing the Set Covering Model provides a robust foundation for modeling and solving strategic sourcing problems involving contract bidding. Its flexibility allows for the inclusion of various operational constraints and objectives. Continued research is essential to enhance scalability, incorporate dynamic elements, and align with modern sustainability goals.

MATERIALS AND METHODS

Problem Formulation

One of the problems facing Nigeria Air Force in times of contracts and procurement of materials includes risk mitigation, inaccurate data and supply related issues. The Set Covering Problem (SCP) postulates that, given a finite set U and a family S of subsets of U, the goal is to find a minimum-cost subfamily of S, referred to as a "cover," $\subset S$, such that the union of all the sets in C is U. In other words given a collection of elements, the set covering problem aims to find the minimum number of sets that incorporate (cover) all of these elements.

Assuming that each $s\epsilon S$ incurs a fixed cost(s) the SCP can be formulated as follows:

$$\text{SCP: Minimize } Z = \sum_{s\epsilon S} C(s)X_s \qquad (1)$$

PO

$$\text{subject to } \sum_{s\epsilon S/u\epsilon S} X_s \geq 1, \quad u\epsilon U \qquad (2)$$

$$X_s \epsilon \{0,1\}, \forall\ s\epsilon S \qquad (3)$$

In this formulation, equation (1) minimizes the total cost of the cover, (2) ensures every element in the original set U is covered by at least one subset in the cover, and (3) describes that every subset either is in the cover or not. Therefore, in this study, an optimization of strategic sourcing using set covering problem formulation has been used in order to select the best bid among offerors, for NAF procurement problem.

In using the set covering formulation, the notations shown in table 1 and cases of misclassification are used.

Table 1. Notations used

Symbol	Description
$\boldsymbol{I}$	set of installations, for iϵI
C	set of offerors (contractors), for cϵC
B	set of bids, for bϵB
$B_i \subseteq B$	subset of bids which contain installation i
$I_b \subseteq I$	subset of installations in bid b
$c_b \in C$	offeror for bid b
P_b	price of bid b [₦]
v_{c_b}	performance rating of offeror c for bid b [rating] (the lower the rating, the better the performance)
w	penalty weight of performance with respect to cost [₦/performance rating]
h_i	penalty factor to reflect importance of having a good performance offeror for installation i [multiplicative factor]
x_b	binary decision variable: 1 if bid b is selected, and 0 otherwise

Set covering formulation

The strategic sourcing for pricing of bids submitted by technically acceptable offerors on multiple installations can be modeled as an SCP, described by equations (1). In this case, the universal set consists of all the bids single, as well as multiple contract types as explained in the previous sections. For example, consider offerors A and B bidding for a certain service to be performed at installations 1, 2, and 3. Table 2 lists all the possible bids by these offerors on all the three installations. For example, Bid #1 is a bid offered by A on Installation 1 alone, whereas Bid #6 is a bid offered by A on Installation 1 and Installation 3, and Bid #7 offers the same service for the three installations clustered together. There are 14 such possible bids. However, in reality, all offerors may not bid on all possible bids due to their own preference or conditions imposed by the Air Force. One such stipulation may be maximum installations allowed to be included in a single bid, which would be a parameter in our model. The principle underlying this strategy of bidding is that the more installations are included in a bid by the offeror, the more the discount in price due to, for example, economies of scale or geographic proximity. In other words, the sum of individual prices in Bids #1 and #2 for Installations 1 and 2 individually considered, respectively, is higher than the pricing in Bid #4 for Installations 1 and 2 included in a single bid. More generally, let b denote a bid for a group of installations I_b, and let P_b be its price. We will assume the following "triangular" relationship holds:

If $I_b \cup I_{b'} = I_{b''}$, then, for any offeror bidding for I_b , $I_{b'}$, and $I_{b''}$, $P_b + P_{b'} > P_{b''}$.

Note that, if the above is not true, we may trivially eliminate bid b" from the pool. In fact, the above may be generalized to mixed contractors and bids such that $I_b \cup I_{b'} \supseteq I_{b''}$. That is, if offerors A, B and C bid on I_b , $I_{b'}$, and $I_{b''}$, respectively, we may eliminate the third bid if its price exceeds the sum of the other two.

Table 2. List of Possible Bids

Offeror	Bid number	Installation 1	Installation 2	Installation 3
A	1	X		
	2		X	
	3			X
	4	X	X	
	5		X	X
	6	X		X
B	7	X	X	X
	8	X		
	9		X	
C	10			X
	11	X	X	
	12		X	X
	13	X		X
	14	X	X	X

The decision is which bids should be selected in order to achieve the pre-set strategy set by the authority. The strategy might be to choose those bids that enjoy the most confidence in performance level (CPL) without any consideration to the cost, or the bids that are least expensive with no consideration to CPL. However, common sense dictates that in most cases, the strategy will be a compromise between these two objectives. The optimization model will account for this compromise by incorporating a weight as its input. The objective of the model is to achieve this pre-set strategy subject to the fundamental constraint that all installations receive the service.

Assumptions

The following would be the tentative underlying assumptions, based on discussions with subject-matter experts. However, all these assumptions may be adjusted by individual program managers as they apply the model. For example, in a scenario we may assume the maximum number of installations an offeror can bid on simultaneously is five, but this number could be different for different offerors. The following assumptions will solely be used for ease in developing the scenarios:

1. Each Offeror bids on numerous bids, but the maximum number of installations, n , in a bid is fixed.
2. All Offerors offer the same percentage of quantity discounts that are based on number of installations included in the bid.
3. All installations have the same preference in CPL of the Offerors.

In developing this model, the notations in Table 1 are used for optimal offeror and bidding selection in the following SCP model:

$$\min z = \Sigma(P_b + wv_{cb}\Sigma h_i)x_b \quad (4)$$

$$subject\ to\ \Sigma x_b \geq 1, \forall\ i \in I \quad (5)$$

$$x_b \in \{0,1\}, \quad \forall b \in B \quad (6)$$

The following tables contain the data for the existing trial and guessing for finding suitable bidding in NAF. Table 3 contain the results of single bids by Offeror per Installation. The table is showing each contractor against what he/she bided for. Table 4 shows the contractors with lowest cost bids. Table 5 is the summary of the lowest costs. While Table 6 shows contractors with best confidence performance level (CPL) and lowest price. Table 7 gives the summary of Table 6, Table 10 gives the list of contractors with combined bids, and then Table 11 gives the price of combined bids per contractor.

Table 3. Results of Single Bids by offeror per Installation

INSTALATION	OFFEROR																
	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OAA 7	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17
NAF 1		723,485	650,125		627,569							925,684			823,186	715,889	
NAF 2		237,556	215,445						199,064				208,565				
NAF 3	298,565	286,125	245,369			398,565		456,000			241,635		237,169	421,882			
NAF 4		917,634	925,618	921,658	952,325		932,548		928,546				930,584	948,687	942,685		
NAF 5	1,309,276	1,425,608	1,350,874										1,625,897	2,148,562	1,526,512		
NAF 6		156,354	175,894	2 25,789									250,325		113,274		
NAF 7			408,996	424,608	375,000		364,860								384,509		
NAF 8			278,996	292,115	262,395	268,975					250,976				265,128		
NAF 9				817,780			882,285		837,601	825,883			850,316		905,112		
NAF 10	582,403	592,445	585,226	601,298	587,497	592,668	592,235						658,988	985,236			
NAF 11			579,446										602,555		602,595		
NAF 12	495,784					492,961			508,556								585,365
NAF 13												832,564			548,126	19,762	

Table 3 above show the prices of a single bid on an installation with OAA 1, OAA 2, OAA 3,…, OAA 17 as the Offerors and NAF 1, NAF 2, NAF3,……, NAF 13 as the Installations.

Selection 1: Lowest Cost

Current processes of selection (before applying the model) are based on two distinct principles. The first process of selection (“Selection 1: Lowest Cost”) chooses the least expensive single bid for an installation. This process parallels the procurement process with emphasis on lowest cost.

Table 4. Selection 1: Lowest Cost

INSTALATION	OFFEROR																
	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OAA 7	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17
NAF 1		723,485	650,125		627,569							925,684			823,186	715,889	
NAF 2		237,556	215,445						199,064				208,565				
NAF 3	298,565	286,125	245,369			398,565		456,000			241,635		237,169	421,882			
NAF 4		917,634	925,618	921,658	952,325		932,548		928,546				930,584	948,687	942,685		
NAF 5	1,309,276	1,425,608	1,350,874										1,625,897	2,148,562	1,526,512		
NAF 6		156,354	175,894	2 25,789									250,325		113,274		
NAF 7			408,996	424,608	375,000		364,860								384,509		
NAF 8			278,996	292,115	262,395	268,975					250,976				265,128		
NAF 9				817,780			882,285		837,601	825,883			850,316		905,112		
NAF 10	582,403	592,445	585,226	601,298	587,497	592,668	592,235						658,988	985,236			
NAF 11			579,446										602,555		602,595		
NAF 12	495,784					492,961			508,556								585,365
NAF 13												832,564			548,126	19,762	

Table 5. Summary of Lowest Cost

INSTALATION	OFFEROR	PRICE
NAF 1	OAA 5	627,569
NAF 2	OAA 9	199,064
NAF 3	OAA 13	237,169
NAF 4	OAA 2	917,634
NAF 5	OAA 1	1,309,276
NAF 6	OAA 15	113,274
NAF 7	OAA 7	364,860
NAF 8	OAA 11	250,976
NAF 9	OAA 4	817,780
NAF 10	OAA 1	582,403
NAF 11	OAA 3	579,446
NAF 12	OAA 6	492,961
NAF 13	OAA 16	19,762
TOTAL		6,512,174

Selection 2: Best CPL and Lowest Cost

The second process of selection ("Selection 2: Best CPL and Lowest Cost") first chooses the offeror with the best CPL for that installation. If there is a tie, then it is broken based on cost. Whoever offers the least cost is selected. This selection process is presented in Table 6 below and the summary is in Table 7

Table 6. Selection 2: Best CPL and Lowest Cost

INSTALATION	OFFEROR																
	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OAA 7	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17
NAF 1		723,485	650,125		627,569							925,684			823,186	715,889	
NAF 2		237,556	215,445						199,064				208,565				
NAF 3	298,565	286,125	245,369			398,565		456,000			241,635		237,169	421,882			
NAF 4		917,634	925,618	921,658	952,325		932,548		928,546				930,584	948,687	942,685		
NAF 5	1,309,276	1,425,608	1,350,874										1,625,897	2,148,562	1,526,512		
NAF 6		156,354	175,894	225,789									250,325		113,274		
NAF 7			408,996	424,608	375,000		364,860								384,509		
NAF 8			278,996	292,115	262,395	268,975					250,976				265,128		
NAF 9				817,780			882,285		837,601	825,883			850,316		905,112		
NAF 10	582,403	592,445	585,226	601,298	587,497	592,668	592,235						658,988	985,236			
NAF 11			579,446										602,555		602,595		
NAF 12	495,784					492,961			508,556								585,365
NAF 13												832,564			548,126	19,762	
CPL	1	1	1	3	2	3	3	5	3	5	1	1	3	1	2	3	5

Table 7. Summary of Selection 2

INSTALATION	OFFEROR	PRICE
NAF 1	OAA 3	650,125
NAF 2	OAA 3	215,445
NAF 3	OAA 11	241,635
NAF 4	OAA 2	917,634
NAF 5	OAA 1	1,309,276
NAF 6	OAA 2	156,354
NAF 7	OAA 3	408,996
NAF 8	OAA 11	250,976
NAF 9	OAA 15	905,112
NAF 10	OAA 1	582,403
NAF 11	OAA 3	579,446
NAF 12	OAA 1	495,784
NAF 13	OAA 12	832,564
TOTAL		7,545,750

In order to demonstrate the strategic sourcing concept using economies of scale for this study, we have used the single bids to create combined bids using the following two rules:

1. Each offeror combines individual bids (to create multi-installation bids) up to a maximum number of installations per bid, n. In our examples, we set $n = 5$. That is, if an offeror initially has $m \geq 5$ individual bids, we will add new bids combining 2, 3, 4, and 5 of those bids, respectively, that is, a total of $\binom{m}{2} + \binom{m}{3} + \binom{m}{4} + \binom{m}{5}$ bids. Of course, if the offeror has $m < 5$ individual bids, then we only generate $\binom{m}{2} + \binom{m}{3} + \cdots + \binom{m}{m}$ new combined bids $\left(\text{where } \binom{m}{r} = \frac{m!}{(m-r)!r!}\right)$. The result of combine bids is represented in Table 10 below.
2. All offerors offer the same percentage of quantity discounts, which are based on the number of installations combined in the bid. Specifically, we set the discount rate rk offered by any offeror who is awarded k installations simultaneously to 2, 5, 8, and 10% for k=2,3,4, and 5 installations, respectively. We also assign numerical values to CPL for utilizing the mathematical model solely for ease in developing the scenarios which are given in Table 9 and Table10 shows the discounted price by each offeror per number of installations.

Table 8. Numerical Values for CPL

Substantial Confidence	1
Satisfactory Confidence	2
Unknown Confidence	3
Limited Confidence	4
No Confidence	5

Based on the given category of confidence in performance levels (such as substantially confident or not confident, for instance), and the numerical scale described in Table 6 each offeror was assigned a numerical value for its CPL. These are listed in Table 8. The smaller the value of CPL, the better the confidence in the performance level.

Table 9. Numerical Values of CPL for Offerors

Offerors	CPL
OAA5	2
OAA10	3
OAA14	3
OAA2	1
OAA1	1
OAA16	2
OAA7	3
OAA12	1
OAA4	3
OAA15	1
OAA3	1
OAA6	3
OAA17	3

Table 10. Combined Bids by offeror

	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OM1	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17	Total
Single bid	4	7	10	6	5	4	4	1	4	1	2	2	8	4	9	2	1	74
2 bids	6	21	45	15	10	6	6	0	6	0	1	1	28	6	36	1	0	188
3 bids	4	35	120	20	10	4	4	0	4	0	0	0	56	4	84	0	0	345
4 bids	1	35	210	15	5	1	1	0	1	0	0	0	70	1	126	0	0	466
5 bids	0	21	252	6	1	0	0	0	0	0	0	0	56	0	126	0	0	462
Total number of combine bid	11	112	627	56	26	11	11	0	11	0	1	1	210	11	372	1	0	1,461
Total bids	**15**	**119**	**637**	**62**	**31**	**15**	**15**	**1**	**15**	**1**	**3**	**3**	**218**	**15**	**381**	**3**	**1**	**1,535**

From Table **10** above we can observed that:

I. There are 74 single bids
II. There are 188 double bids (2 bids) which were calculated using the formula $\sum\binom{n}{2}$, where n is the total number of single bids
III. There are 345 triple bids (3 bids) which were calculated using the formula $\sum\binom{n}{3}$, where n is the total number of single bids
IV. There are 462 fourth bids (4 bids) which were calculated using the formula $\sum\binom{n}{4}$, where n is the total number of single bids
V. There are 462 fifth bids (5 bids) which were calculated using the formula $\sum\binom{n}{5}$, where n is the total number of single bids
VI. There are 1461 combined bids (sum of 2 bids, 3 bids, 4 bids and 5 bids)
VII. There are 1535 total bids (sum of single bids, 2 bids, 3 bids, 4 bids and 5 bids)

Price of combined bids

To find the prices of the combined bids per number of installations, we use the following:

I. For combined two bids per offeror, the price of the bid is

$$P_b = \frac{98(n-1)\sum_{i=1}^{n} P_i}{2(100)l},$$

where n is the total number of single bids, l is the number of two bids and P_i is the price of each combine bid.

II. For combined three bids per offeror, the price of the bid is

$$P_b = \frac{95(n-1)\sum_{i=1}^{n} P_i}{3(100)l},$$

where n is the total number of single bids, l is the number of three bids and P_i is the price of each combine bid.

III. For combined two bids per offeror, the price of the bid is

$$P_b = \frac{92(n-1)\sum_{i=1}^{n} P_i}{4(100)l},$$

where n is the total number of single bids, l is the number of four bids and P_i is the price of each combine bid, while

IV. For combined five bidsper offeror, the price of the bid is

$$P_b = \frac{90\left(\binom{n}{5}-\sum\binom{n-1}{5}\right)\sum_{i=1}^{n} P_i}{5(100)l},$$

where n is the total number of single bids, l is the number of five bids and P_i is the price of each combine bids.

The result of price of combine bids is shown in Table 11 below.

The formular was derived from a percentage discount by each offerors, considering the number of bids bided for by each contractor or offeror. For example we set n=5

That is, if an offeror initially has $m \geq 5$ individual bids, we will add new bids combining 2, 3, 4, and 5 of those bids, respectively, that is, a total of $\binom{m}{2} + \binom{m}{3} + \binom{m}{4} + \binom{m}{5}$ bids. Of course, if the offeror has $m < 5$ individual bids, then we only generate $\binom{m}{2} + \binom{m}{3} + \cdots + \binom{m}{m}$ new combined bids $\left(\text{where } \binom{m}{r} = \frac{m!}{(m-r)!r!}\right)$.

Table 11. Price of Combined Bids per Offeror (₦)

	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OM1	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17
2 bids	658,077	607,489	530,767	499,385	549,738	429,526	679,122	0	606,073	0	241,379	861,542	657,139	1,103,570	665,434	360,469	0
3 bids	637,932	235,557	128,630	242,049	355,273	416,378	658,333	0	587,520	0	0	0	212,341	106,9787	184,304	0	0
4 bids	617,786	114,059	41,523	140,643	258,040	403,229	637,543	0	568,966	0	0	0	88,129	1,036,004	66,931	0	0
5 bids	0	520,705	1,137,358	458,619	504,861	0	0	0	0	0	0	0	482,796	0	366,668	0	0
Pb	478,449	369,452	459,569	335,174	416,978	312,283	493,750	0	440,640	0	60,345	215,385	360,101	802,340	320,834	90,117	0

Table 11 describes cost for each offeror per 2 bids, 3 bids, 4 bids, 5 bids and total combined bids. It can be observed from the table that:

1. Offerors OAA 8 and OO17 has ₦0 for both 2 bids, 3 bids, 4 bids, 5 bids and total combined bids because they only have one single bid from Table 12 above.
2. For any offeror who has more than four single bidhe/she will have prices for 2 bids, 3 bids, 4 bids, 5 bids and total combined bids.

The model in (4 to 6) is reproduce here for smooth flow of ideas for optimal offeror and bidding selection as the following SCP model:

$$Min\ z = \sum(P_b + wv_{cb}\sum h_i)x_b$$

$$\text{subject to} \quad \sum x_b \geq 1, \quad \forall \ i \in I$$

$$x_b \in \{0,1\}, \quad \forall\ b \in B$$

To obtain the constraint of our PO model, the constraints are transformed to a binary code and the result is in Table 13 below.

Table 12. Binary decision variables

	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OM1	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17
NAF 1	0	1	1	0	1	0	0	0	0	0	0	1	0	0	1	1	0
NAF 2	0	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0
NAF 3	1	1	1	0	0	1	0	1	0	0	1	0	1	1	0	0	0
NAF 4	0	1	1	1	1	0	1	0	1	0	0	0	1	1	1	0	0
NAF 5	1	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0
NAF 6	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	0
NAF 7	0	0	1	1	1	0	1	0	0	0	0	0	0	0	1	0	0
NAF 8	0	0	1	1	1	1	0	0	0	0	1	0	0	0	1	0	0
NAF 9	0	0	0	1	0	0	1	0	1	1	0	0	1	0	1	0	0
NAF 10	1	1	1	1	1	1	1	0	0	0	0	0	1	1	0	0	0
NAF 11	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0
NAF 12	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1
NAF 13	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0

Table 12 shows the binary decision variable x_b of the PO model. It can be observed that;

$$x_b = \begin{cases} 1 & if\ bid\ \ b\ is\ selected \\ 0 & otherwise \end{cases} \tag{7}$$

Table 13. Results of the penalty factor to reflect importance of having a good cost

Hi	OAA 1	OAA 2	OAA 3	OAA 4	OAA 5	OAA 6	OM1	OAA 8	OAA 9	OAA 10	OAA 11	OAA 12	OAA 13	OAA 14	OAA 15	OAA 16	OAA 17	
NAF 1		1.00	1.00		0.50							0.20			0.50	0.33		3.53
NAF 2		1.00	1.00						0.33				0.33					2.67
NAF 3	1.00	1.00	1.00			0.33		0.25			1.00		0.33	0.20				5.12
NAF 4		1.00	1.00	0.33	0.50		0.33		0.33				0.33	0.20	0.50			4.53
NAF 5	1.00	1.00	1.00										0.33	0.20	0.50			4.03
NAF 6		1.00	1.00	0.33									0.33		0.50			3.17
NAF 7			1.00	0.33	0.50		0.33								0.50			2.67
NAF 8			1.00	0.33	0.50	0.33					1.00				0.50			3.67
NAF 9				0.33			0.33		0.33	1.00			0.33		0.50			2.83
NAF 10	1.00	1.00	1.00	0.33	0.50	0.33	0.33						0.33	0.20				5.03
NAF 11			1.00										0.33		0.50			1.83
NAF 12	1.00					0.33			0.33								0.20	1.87
NAF 13												0.20			0.50	0.33		1.03
Total	4.00	7.00	10.00	2.00	2.50	1.33	1.33	0.25	1.33	1.00	2.00	0.40	2.67	0.80	4.50	0.67	0.20	

Table 13 shows the penalty factor which reflects importance of having a good performance offeror for installation i [multiplicative factor] and it is calculated using the following formula

$$h_i = \frac{1}{(1-[\mathrm{CPL}_i - 1])} \tag{8}$$

where CPL_i is the confidence performance level of ith unit.

The formula hi was derived by considering the penalty factor to reflect importance of having a good performance offeror for installation i (multiplicative factor).

Now in our effort to improve the existing trial and guessing of the existing manual way of taking bids considering equations (4), (5)and (6), we have the following optimization results shown in Table 14 below.

Method

The method was programmed in MATLAB programming language and the results were evaluated with an **Optimization SolverLINGO**. These techniques were implemented using a benchmark problem gotten from the literature. However, it was observed that LINGO being an optimization solver performs better.

RESULTS AND DISCUSSION

An improved solution using Set Covering Problem

Table 14. Results of the optimization model using the PO model for various combined bids

Number of bids	Total Optimal price			
	Scenario-1 ($w = 1{,}000$)	Scenarios-2 ($w = 1{,}500$)	Scenarios-3 ($w = 2{,}000$)	Scenarios-4 ($w = 2{,}500$)
2 bids	₦2,923,610.94	₦ 3,458,277.61	₦ 3,992,944.28	₦ 4,527,610.94
3 bids	₦2,020,198.55	₦ 2,554,865.22	₦ 3,089,531.89	₦ 3,624,198.55
4 bids	₦1,795,573.75	₦ 2,330,240.41	₦ 2,864,907.08	₦ 3,399,573.75
5 bids	₦2,573,358.64	₦ 3,108,025.31	₦ 3,642,691.98	₦ 4,177,358.64
combined bid	₦2,328,185.47	₦ 2,862,852.14	₦ 3,397,518.80	₦ 3,932,185.47

The first selection process (similar to procurement process with emphasis on lowest cost) yields a cost of ₦ 6,512,174, and the second selection process (similar to PPT process) yields a cost of ₦7,545,750, 5 The second selection process which prioritizes the offeror's CPL and then the least expensive yields an average TCPL of about 1.15 per installation (translating to slightly less than Substantial Confidence in performance), but this occurs at an extra cost of 749,138. The results of the implementation of the PO model in Scenarios 1 through 4 (scenarios named for different values of weight function w) are described in Table 14, the cost varies from ₦1,795,573.75 to ₦4,527,610.94, which shows the effect or significance of introducing the weight function in the PO model. The solution from the PO model (Model-Scenario-3) is cheaper than the current process (Selection 1) solution by more than ₦4,000,000. On the other hand, in terms of number of bids, the best average least cost is in a case of 4 bids in all the scenarios (Scenario-1 has ₦1,795,573.75, Scenario-2 has ₦ 2,330,240.41, Scenario-3 has ₦ 2,864,907.08 and Scenario-4 has ₦ 3,399,573.75 cost) and the highest cost is in the case of 2 bids in all the scenarios (Scenario-1 has ₦2,923,610.94, Scenario-2 has ₦ 3,458,277.61, Scenario-3 has ₦ 3,992,944.28 and Scenario-4 has ₦ 4,527,610.94 cost). From Table 15 above, it can be observed that as the number of the weight value increases, the optimal cost also increases across all the scenarios. The information of Table 14 is represented in Figure 1 below;

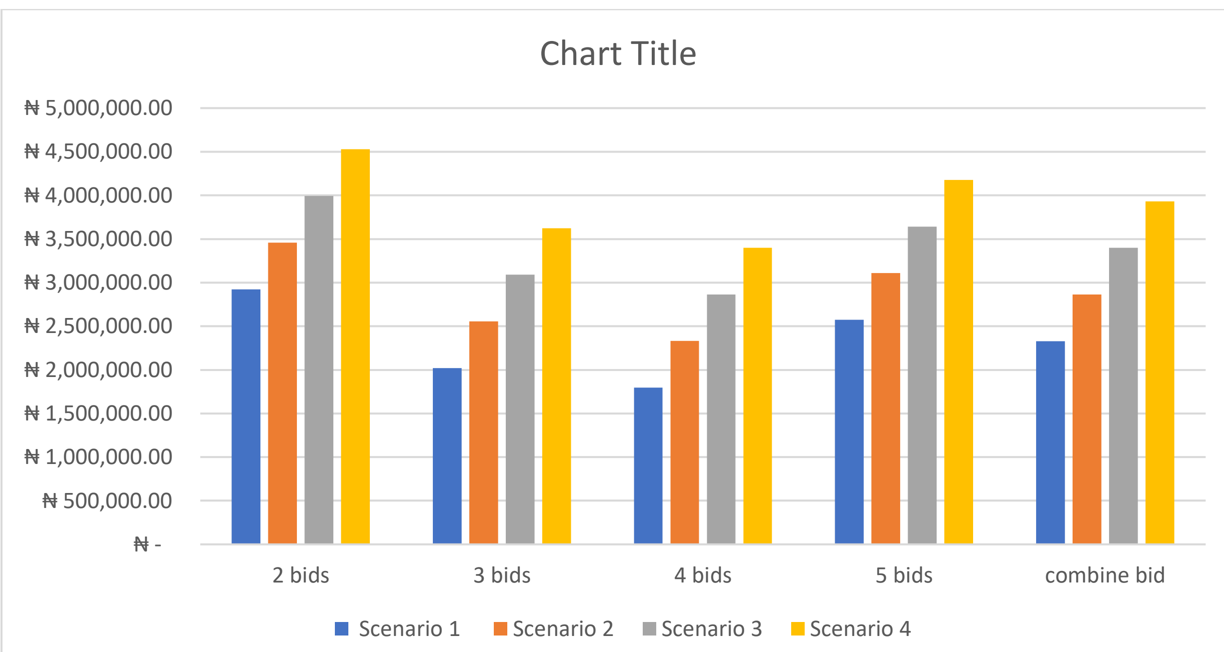

Figure 1. Bar Chart representing various scenario

CONCLUSION

In the research work, it can be observed that the first selection process (similar to procurement process with emphasis on lowest cost) yields a cost of ₦ 6,512,174, and the second selection process (similar to PPT process) yields a cost of ₦7,545,750. The second selection process which prioritizes the offeror's CPL and then the least expensive yields an average TCPL of about 1.15 per installation (translating to slightly less than Substantial Confidence in performance), but this occurs at an extra cost of 749,138. The results of the implementation of the Price optimization model in Scenarios 1 through 4 (scenarios named for different values of weight function *w*) are described in Table 14. The cost varying from ₦1,795,573.75 to ₦4,527,610.94. The solution from the PO model (Model-Scenario-3) is cheaper than the current process (Selection 1) solution by more than ₦4,000,000. On the other hand, in terms of number of Bids, the best average least Cost is in a case of 4 bids in all the scenarios (Scenario-1 has ₦1,795,573.75, Scenario-2 has ₦ 2,330,240.41, Scenario-3 has ₦ 2,864,907.08 and Scenario-4 has ₦ 3,399,573.75 cost) and the highest cost is in the case of 2 bids in all the scenarios (Scenario-1 has ₦2,923,610.94, Scenario-2 has ₦ 3,458,277.61, Scenario-3 has ₦ 3,992,944.28 and Scenario-4 has ₦ 4,527,610.94 cost).

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