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Modelling and Forecasting Long Memory and Volatility in Nigeria's Consumer Price Index Using an Arfima-Figarch Approach

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ABSTRACT
Commodity prices in Nigeria exhibit long memory characteristics, which lead to high risk of price fluctuations. This study aims to model and forecast the impact of long memory on cereal prices index in Nigeria using a hybrid time series model. The data used for this study are secondary monthly CPI data obtained from the Central Bank of Nigeria (CBN) covering the period 1990-2025. The study employed Kwiat-Kowski Phillip Smidth Shin test to check for stationarity and found that the variables were stationary after taken fractional differencing. The study employed GPH test to check for long memory in the variables and it was found in the variables. The study utilized scatter plot to check for heteroscedasticity and it was found in the residuals of ARFIMA (1, 1.2, 1) models. In addition, ARFIMA (1, 1.2, 1)-FIGARCH (9, 1) was found as the best model with least AIC, MAE, MSE, and RMSE when compared with standalone models. The study employed Ljung-Box and ARCH-LM tests to diagnose the models. Hence, there is no excess correlation in the residuals of best models. The forecast results shows that the forecasted volatility of the CPI variable increases over time, this indicates the rising prices may impact food affordability and accessibility in the nation. Moreover, Government may need to intervene to stabilize prices and ensure food security.

Keywords:
ARFIMA,
FIGARCH,
CPI,
Long Memory,
Volatility,
Heteroscedasticity

INTRODUCTION

Long memory means past values in a series influence future values over long time spans, causing slow decay in autocorrelations (Sottinen, 2021). This phenomenon is commonly observed in stock returns and particularly volatility, challenging the Efficient Market Hypothesis (EMH) which assumes price changes are random (Kramer, 2001). Several Studies have confirmed that long memory allows for predictability in returns and volatility, implying trading strategies can exploit persistence trends and risk (Barunik and Kristoufek, 2019). Capturing long memory is thus crucial for accurate forecasting and risk management.

Nigeria's commodity market is highly volatile and influenced by macroeconomic and monetary variables like money supply, interest, and exchange rates, affecting stock price movements (Egberi and Olsufolan, 2021). Despite its growing importance, persistent dependencies and long-run dynamics in Nigerian commodity prices remain underexplored. Understanding these dynamics helps investors and policymakers mitigate risks and enhance market efficiency amid Nigeria's unique economic conditions.

Hybrid time series models combine strengths of different methods to capture linear and nonlinear patterns in financial data more accurately (Stempien and Slepaczuk, 2013). For example integrating ARFIMA for long memory with FIGARCH for volatility models has proven robust in forecasting and modelling complex dependencies in other markets. Such hybrids outperform single models by adapting to changing market conditions and capturing complex behaviours.

Studying long memory enables better understanding of price persistence, improving forecasts and trading strategies. It reveals market inefficiencies contradicting EMH, informing regulatory and investment decisions. Long memory analysis aids in valuing stock options and risk management by modelling volatility's persistence. For Nigeria leveraging this can enhance market predictability and investor confidence.

Studies were conducted by various researchers with the use of ARFIMA-FIGARCH model on economic data. These include Benbachir (2025) assessed informational efficiency in largest African stock markets by modelling dual long memory using ARFIMA-FIGARCH approach.

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The study assessed the weak-form informational efficiency of six major African stock markets – Johannesburg, Casablanca, Botswana, Nigeria, Egypt, and the Regional Stock Exchange – through the lens of long-memory behavior in returns and volatility. This was achieved by employing four advanced models: ARFIMA-FIGARCH, ARFIMA-FIEGARCH, ARFIMA-FIAPARCH, and ARFIMA-HYGARCH. Each of these models was specifically designed to capture long memory in both the conditional mean and variance. The empirical results demonstrated that the ARFIMA- FIGARCH framework, across all four model variants, consistently outperformed alternative specifications in fitting the return and volatility dynamics of all six African stock market indices. Moreover, Zhelyazkova (2018) studied exchange rates using ARFIMA-FIGARCH, HYGARCH and FIAPARCH Models. The used daily exchange rate returns of twelve currencies against USD (4310 observations) from 2000 to 2017. The tests were based on ARFIMA-FIGARCH, HYGARCH and FIAPARCH models which are estimated by maximum likelihood method under the assumption of t-distribution, generalized error distribution and skewed t-distribution of innovation process. The results showed presence of long memory in volatility of all twelve exchange rates and dual long memory in the returns of BRL/USD only. The HYGARCH model was found to be an appropriate volatility model with long memory for BRL/USD, NOK/USD and ZAR/USD. According to estimated FIAPARCH models there is an asymmetric response of volatility of BRL/USD, MXN/USD, NZD/USD and ZAR/USD to positive and negative shocks along with long memory. Furthermore, Turkyilmaz and Balibey (2014) studied Long Memory Behavior in the Returns of Pakistan Stock Market using ARFIMA-FIGARCH model. The data used for the study consists of daily stock index data for the period of 2010-2013 after global economic crisis. According to findings of the study, ARFIMA model do not support long memory behavior for the stock market returns. However, FIGARCH model indicated that volatility of market returns has long memory. Moreover, in order to test the feature of long memory in the return and volatility of the stock market simultaneously, ARFIMA-FIGARCH models were estimated according to different distributions simultaneously. Predictable structure of volatility of Pakistan stock market displayed that this market was the weak-form market inefficiency. Consequently, it was possible to say that technical analysis related to this stock market may be valid. This implied that it was possible to predict future stock prices and extra ordinary gains could be obtained trading in the market.

Despite the importance of cereal commodity in Nigeria's economy, there is a death of research on modelling and forecasting its prices index using robust time series techniques. Specially, existing literatures are scarce on studies that apply advanced time series models to capture the complex dynamics of these commodity prices. Notably, there is a lack of research that employs hybrid time series models that simultaneously account for long memory in both the mean process and volatility of these price indices. To address these gaps, the study proposes to employ ARFIMA-FIGARCH model to examine the prices index of cereal commodity in Nigeria, providing a more comprehensive understanding of its behaviour and contributing to the limited literature on this research.

The study aims to model and forecast the impact of long memory properties on Nigerian commodity prices index using ARFIMA-FIGARCH approach. And the objectives are to: (i) Analyse long memory characteristics in Nigeria's commodity prices index data (ii) Estimate and compare parameters capturing persistence in returns and volatility (iii) Evaluate the forecasting performance of the model for commodity price index volatility (iv) Assess the implication of long memory on market efficiency and risk management in Nigeria. Hence our focal research questions are: (i) Does the Nigeria's commodity prices index exhibit long memory in returns and volatility? (ii) How well does ARFIMA-FIGARCH model capture these long memory features? (ii) Can the model improve forecasting accuracy of the commodity prices index volatility? (iii) Can the model improve forecasting accuracy of the commodity prices index volatility? (iv) What are the implications of long memory on market efficiency and risk management in Nigeria?

MATERIALS AND METHODS

Method of Data Collection

The data collected for this research work are secondary data which was retrieved online through www.c.b.n.gov.ng website. The data is economic time series data which is based on commodity prices index in Nigeria. The data was collected on monthly basis ranging from 1990-2025.

Method of Data Analysis

This section describes the analytical procedures applied in Chapter Four. The methods employed include time series visualization, stationarity testing, long memory testing, model order selection, heteroscedasticity testing, model estimation, diagnostic checking, and forecast performance evaluation using ARFIMA, FIGARCH, and ARFIMA-FIGARCH models.

ARFIMA Model

The Autoregressive Fractional Integrated Generalize Moving Average model was proposed by Granger and Joyeux (1980) and Hosking (1981). The model is used capture the presence of long memory in the time series. The model is defined

$$\emptyset(B)(1-B)^d X_t = \Theta(B)e_t \quad (1)$$

Where,
$\emptyset(B)$: This is the autoregressive operator; it is a set of coefficients ($\varphi_1, \varphi_2,$ ... capturing the effect of past values of the series $X_{t-1}, X_{t-2}, ...$, on the current value (X_t). Where, B is the lag operator ($B(X_t) = X_{t-1}$).
$(1-B)^d$: is the fractional differencing operator. Where, (d) is decimal, not whole number, letting the model to capture long memory, meaning that past shocks still influence the present, but may be decaying slowly over time.
X_t : is the observed time series value at time (t).
$\boldsymbol{\Theta}(B)$: is the moving average operator it is set of coefficients $(\theta_1, \theta_{2,...})$ but applied to past error terms (residuals), that is $\varepsilon_{t-1}, \varepsilon_{t-2}, ...$
ε_t : is the white noise or error term at time (t).

FIGARCH Model

The study employed FIGARCH model developed by Bollerslev and Mikkelsen (1996). The model is an extension or an improvement of GARCH model where the long memory component was added to the model. The model is used to capture characteristic of volatility long memory, persistence and clustering. The model is given below as:

$$\sigma^2{}_t = \omega + \alpha_i \sum_{i=1}^{q} \varepsilon^2{}_{t-i} + \beta_j \sum_{j=1}^{p} \sigma^2{}_{t-j} + \left[1-(1-L)^d\right]\varepsilon^2{}_t \qquad (2)$$

Where, $\sigma^2{}_t$ is the conditional variance, $\varepsilon^2{}_{t-i}$ is the squared unexpected returns for the previous periods, $\sigma^2{}_{t-j}$ is the previous volatility, ω is the intercept, α_i is the adjustment of the past shocks and β_j is the adjustment to the past volatility, L is the lag operator, d is the differencing operator and $\varepsilon^2{}_t$ is the squared of the residuals.

ARFIMA-FIGARCH Model

The study employed ARFIMA-FIGARCH which is a combination of ARFIMA and FIGARCH models. The model addresses the limitations of ARFIMA and FIGARCH models. The ARFIMA model has ability to capture long memory process in mean of a time series; however, it cannot capture long memory process in volatility and heteroscedasticity of a time series. The FIAGRCH model has ability to capture long memory in volatility process and heteroscedasticity; however, it cannot capture long memory process in mean of a time series. Therefore, ARFIMA-FIGARCH model overcome the limitations of both two models. The model is given below as:

$$\sigma_t^2 = \omega + \alpha_i \sum_{i=1}^{q} (\sigma_t z_t)_{t-i}^2 + \beta_j \sum_{j=1}^{p} \sigma_{t-j}^2 + \left[\left(1-(1-L)\right)^d (\sigma_t z_t)_t^2\right] \qquad (3)$$

Where, $\sigma^2{}_t$ is the conditional variance, $(\sigma_t z_t)_{t-i}^2$ is the squared unexpected returns for the previous periods of the ARFIMA model, $\sigma^2{}_{t-j}$ is the previous volatility, ω is the intercept, α_i is the adjustment of the past shocks and β_j is the adjustment to the past volatility, L is the lag operator, d is the differencing operator and is the squared of the residuals.

Assumptions of ARFIMA-FIGARCH Model

i. It assumes stationarity.
ii. It captures mean and variance of a time series.
iii. It captures long memory in mean and volatility.
iv. Captures heteroscedasticity.
v. Captures AR component.
vi. Captures MA component.

Method to fit ARFIMA-FIGARCH Model

This research work used two phrase methods to fit the ARFIMA-FIGARCH model. In phase (I) the residuals of ARFIMA model were extracted. In phase (II) the extracted residuals from the ARFIMA model were used to fit ARFIMA-FIGARCH model.

Time Series Visualization

This study used time series plot to identify the behavior of the stock prices over the long period of time, make inference about the presence of unit roots, and structural breaks in the time series.

Stationarity Checking

Stationarity in time series analysis is presence of constant mean and variance in time series data. This study employed Augmented Dickey Fuller (ADF) and Kwiat-Kowski Smidth-Shin (KPSS) tests to check for the stationarity in the study variables.

Augmented Dickey-Fuller Test

The study employed test for a unit root in the time series by the test which was developed by Said and Dickey-Fuller (1984). The test is defined below as:

$$\tau = \frac{(\beta_1 - d)}{\sqrt{\frac{\beta_2^2 + \sigma^2}{(1-\beta^2)^2}}} \qquad (4)$$

Where, β_1 is the trend term, d is the differencing parameter, β_2 is the coefficient on the lagged first difference term, and σ^2 is the variance of the time series.
The test involves the following hypotheses:
H_0: the time series has unit roots.
H_a: the time series non-unit roots.

Decision Criteria

Null hypothesis is rejected if P-value is less than the alpha value.

KPSS Test

This research work checked for the presence of unit roots with the test which was proposed by Kwiat-Kowski Smidth-Shin (1992). The test serves as the second approach to check for the unit roots of the time series data. The test is defined below as:

$$KPSST = \frac{\sum_{t=1}^{T}\left(y_t - \hat{\mu} - \hat{\delta}_t\right)^2}{\hat{\sigma}^2 \sum_{t=1}^{T}\left(1 - \frac{t}{T}\right)^2} \quad (5)$$

Where, y_t is the time series, $\hat{\mu}$ is the mean of the time series, $\hat{\delta}_t$ is the estimated trend coefficient, $\hat{\sigma}^2$ is estimated variance of the time series, and T is the sample size.

Long Memory Checking

Long memory is a phenomena when a time series exhibits decay slowly rather than exponential decay. This study employed Gewek and Porter Hudak (GPH) test, proposed in 1983 to estimate and check for the long memory. The test is defined as:

$$\ln\left[I\left(w_j\right)\right] = \beta_0 + \beta_1 \ln\left[4\sin\left(\frac{w_j}{2}\right)\right] + \varepsilon_j \quad (6)$$

Where, $w_j = \frac{2\pi_j}{T}$, $j = 1,2,\ldots,n$, w_j refers to Fourier frequency Transformation $\left(n = \sqrt{T}\right)$ ε_j represents residual of the model, $I\left(w_j\right)$ is a simple periodogram which is defined as:

$$I\left(w_j\right) = \frac{1}{2\pi T}\left|\sum_{i=1}^{T} \varepsilon_t \lambda^{-w_j t}\right| \quad (7)$$

Model Order Selection

This is the process of selecting the exact orders of a time series models. To fit time series models there is need to select the orders of the model. The study used Autocorrelation Function (ACF) plot and Partial Autocorrelation function (PACF) plot to identify the order of the ARFIMA-FIGARCH model.

Autocorrelation Plot

The study will employ autocorrelation plot to identify the order of Moving Average (MA) model to use in the proposed model. The autocorrelation function is given below as:

$$\rho_k = \frac{\theta_k}{\theta_0} \quad (8)$$

Where, ρ_k is the autocorrelation at lag k, k is the chosen lag, θ_k is the covariance at lag k and θ_0 is the variance.

Partial Autocorrelation Plot

The study employed Partial Autocorrelation Function (PACF) plot to identify the order of Autoregressive (AR) model to use in the ARFIMA-FIGARCH model. The function is defined as:

$$\varphi(k) = \frac{[\rho(k) - \sum[\varphi(j).\rho(k-j)]]}{[1 - \sum[\varphi(j).\rho(j)]]} \quad (9)$$

Where, $\varphi(k)$ is the partial autocorrelation at coefficient at lag k, $\rho(k)$ is the autocorrelation coefficient at lag k, $\varphi(j)$ is the partial autocorrelation at lag j, k is the number of lag, and j is the intermediate lag (j = 1 to k – 1).

Heteroscedasticity Checking

Heteroscedasticity refers to the presence of non-constant variance in time series data. The study employed scatter plot to check for the presence of heteroscedasticity in the residuals of ARFIMA model.

Scatter Plot

This research work used scatter plot to check for the presence of heteroscedasticity in the residual, the residuals were plotted against the time. The residuals are on the vertical line and the times are on the horizontal line.

Evaluation Metric Measures

Measures of forecasting accuracy were used in this study to evaluate the performance of the developed model. In this study, Mean Absolute Error (MAE), Mean Square Error (MSE) and, Mean Square Error (RMSE), and Akaike Information Criterion (AIC) are the forecasting performance methods considered.

Mean Absolute Error

Mean Absolute Error was used in this study to measure the average magnitude of the errors produced by the model developed. The test is defined as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|\sigma_t^2 - \hat{\sigma}_t^2\right| \quad (10)$$

Where, n is the sample size, σ_t^2 is the actual variance at time t and $\hat{\sigma}_t^2$ is the estimated variance at time t.

The following steps were undertaken to compute the MAE:

Step1: Calculation of the absolute difference between each actual value and its corresponding forecasting value.

Mean Square Error

This study employed mean squared error to measure the average squared difference between predicted and actual values. The test statistic is given below as:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \hat{\sigma}_t^2)^2 \quad (11)$$

Where, n is the sample size, σ_t^2 is the actual variance at time t and $\hat{\sigma}_t^2$ is the estimated variance at time t.
The following steps were undertaken to compute the MSE:
Step1: Calculation of the difference between each actual value and its corresponding predicted value.

Root Mean Square Error
This study employed Root Mean Square Error to measure the difference between the predicted and actual values. The test statistic is defined as:

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - \hat{\sigma}_t^2)^2} \quad (12)$$

Where, n is the sample size, σ_t^2 is the actual variance at time t and $\hat{\sigma}_t^2$ is the estimated variance at time t.
The following steps were undertaken to compute the RMSE:
Step1: Calculation of the difference between each actual value and its corresponding predicted value.

Akaike Information Criterion
This study employed AIC to evaluate the relative quality of the developed model for the given set of data. The test statistic is given below as:

$$AIC = -2ln(L) + 2k \quad (13)$$

Where, n is the sample size, σ^2 is the variance of the time series, and ε_t^2 is residuals of the time series.

Diagnostic Test
This study employed Ljung-Box test and Arch-Lm test to assess the goodness of the residuals of the selected best model.

Ljung-Box Test
This study checked for presence of serial correlation in the selected best models using a test proposed by Ljung-Box (1978). The following steps were undertaken to carryout Ljung-Box test for the selected best model. The tests is given below as:

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{\tau_k^2}{n-k} \quad (14)$$

Where, n is the number of observations in the time series, k is the particular time lag to checked, M is the number of time lags to be tested, τ_k is the sample autoorrelation function of the kth residuals term.

ARCH-LM Test
This study used the test proposed by Engle (1982) to check for the effect of heteroscedasticity in the residual of the model. The following steps were undertaken to carry out the ARCH-LM test for the residuals of the selected best model. The test is defined as:

$$Q*(m) = T(T+2)\sum_{j=1}^{m}\frac{\hat{P}_j}{T-L} \quad (15)$$

Where, m is the maximum numbers of lags included in the ARCH effect test, $\hat{P}_j$ is the sample Autocorrelation at lag j for the squared time series and T is the number of non-missing values in the data sample.

RESULTS AND DISCUSSION

Time Series Visualization

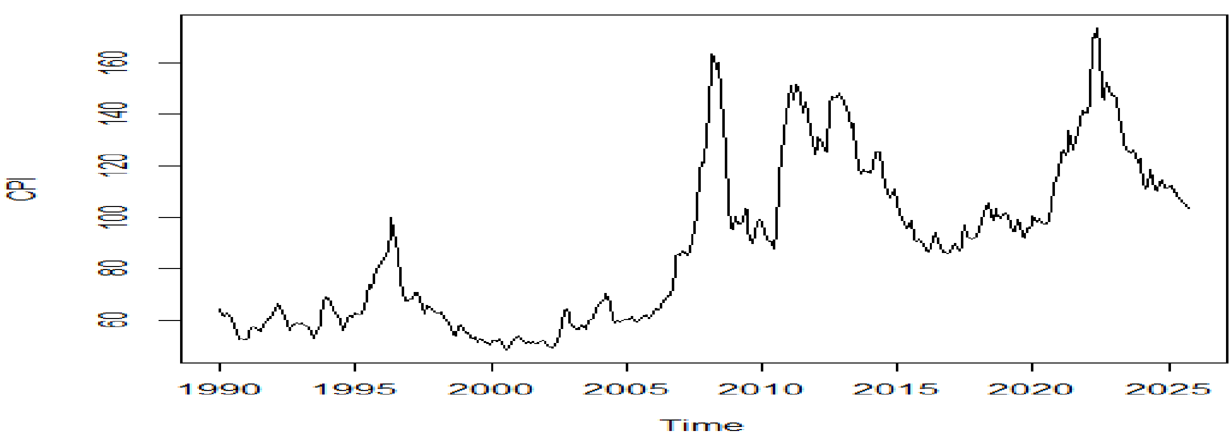

Figure 1 Time Series Plots of Monthly Cereal Price Index (1990-01) to (2025)

From the results obtained in Figure 1 it is observed that the CPI time series exhibits trend both upward and downward movements with a pronounced upward trend, indicating non-stationary behaviour over the study period.

Stationarity Checking

Table 1 ADF Test Results of CPI Time Series

Dickey-Fuller = -2.9701	Lag order = 7	P-value = 0.1676

From Table 1, the ADF p-value (0.1676) exceeds the 5% significant level, hence, the null hypothesis of a unit root cannot be rejected, indicating that the CPI series is non-stationary. However, since the ADF test alone may be inconclusive, the KPSS test was further employed for confirmation.

Table 2 KPSS Test Results of CPI Time Series

KPSS Level = 4.3614	Truncation lag parameter = 5	P-value < 0.01

From the results obtained in Table 2 it is observed that the probability value of KPSS test is less than 0.01, which is less than 0.05, thus, we fail to reject the null hypothesis and conclude that the time series is not stationary.

Long Memory Checking

Table 3 GPH Test of CPI

Estimated (d)	sd.as	sd.reg
1.158523	0.1812318	0.218406

From the results obtained in Table 3 it is observed that the estimated long memory parameter d = 1.1585, which suggest significant long memory in CPI data. This means the series has persistent, slowly fading shocks.

Time Series Visualization of the Differenced CPI

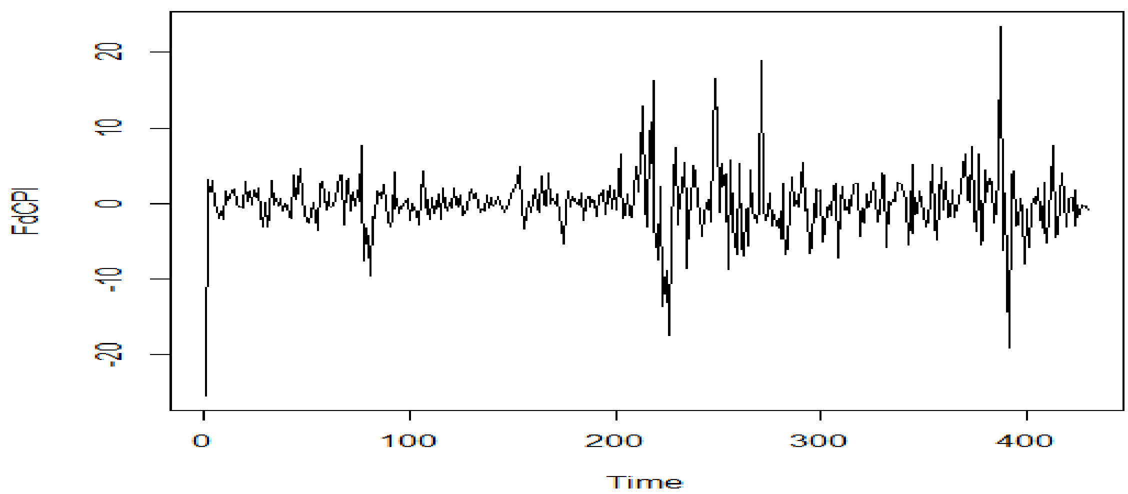

Figure 2 Time Series Plots of the Differenced CPI Time Series

From the results obtained in Figure 2 it is observed that the time series fluctuates around a constant mean (no upward or downward trend). Thus, the time series is stationary.

Stationary Checking of the Differenced Time Series

Table 4 KPSS Test Results of FdCPI Time Series

KPSS Level = 0.028934	Truncation lag parameter = 5	P-value > 0.1

From the results obtained in Table 4 it is observed that the probability value of KPSS test is greater than 0.1, which is greater than 0.05, thus, we are to reject the null hypothesis and conclude that the time series is stationary.

Model Order Estimation

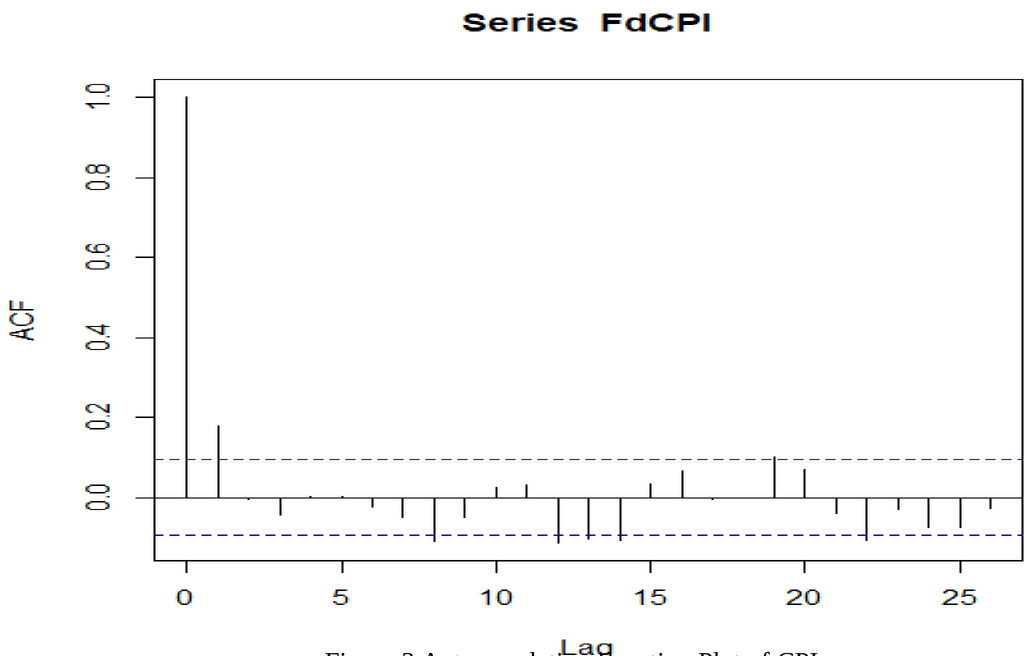

Figure 3 Autocorrelation Function Plot of CPI

From the above results obtained in Figure 3 it is observed that the autocorrelation plot exhibits significant spike at from lag 1 up to lag 2. Thus, MA (1), M

A (2), and MA (9) are significant.

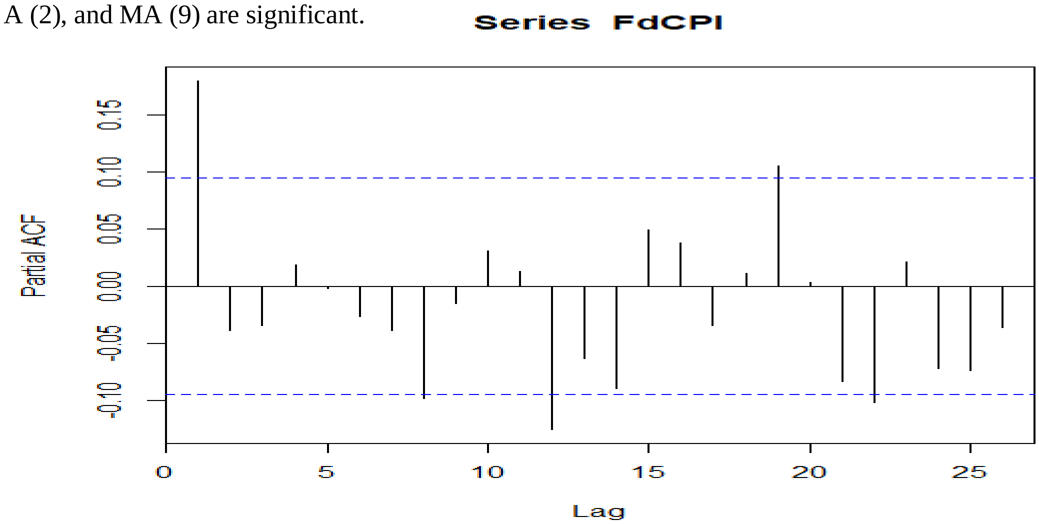

Figure 4 Partial Autocorrelation Function Plot of CPI

From the above results obtained in Figure 4 it is observed that the autocorrelation plot exhibits significant spike at lag. Thus, AR (1) is significant.

Heteroscedasticity Checking in the residuals of ARFIMA (1, 1.2 1) Model

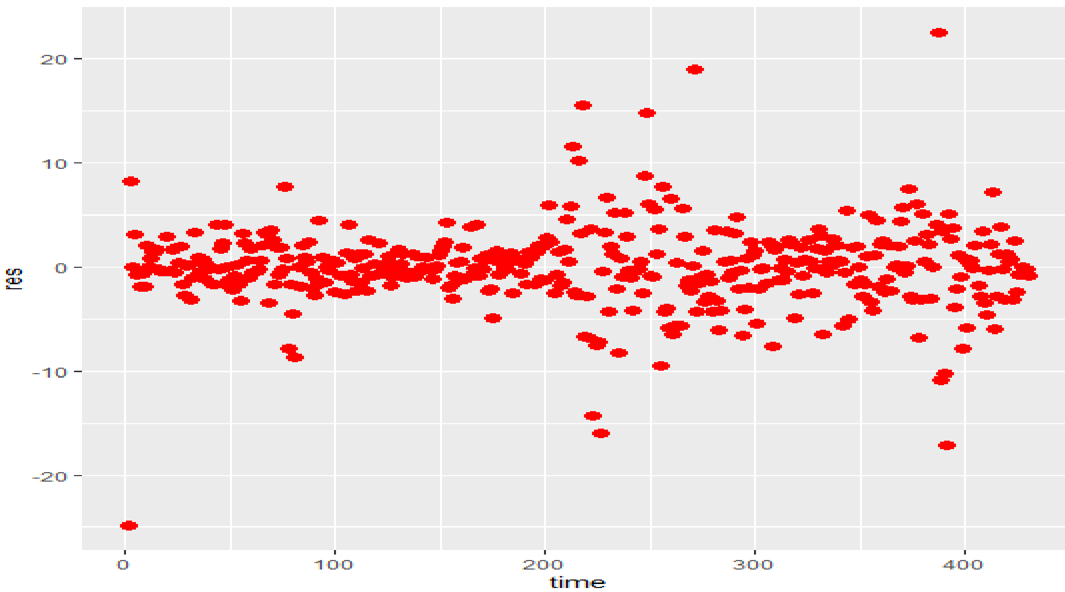

Figure 5 Scatter Plot of ARFIMA (1, 1.2, 1) Residuals

From the results obtained in Figure 5 it is observed that the scatter plot exhibits funnel shape, thus, this indicates presence of heteroscedasticity in the residuals of the model.

Table 5 Comparison of the selected best Models

CPI	ARFIMA (1, 1.2, 1)	2404.19
CPI	FIGARCH (9, 1)	6.3062
CPI	ARFIMA (1, 1.2, 1)-FIGARCH (1, 1)	5.2400

From the results obtained in Table 5 it is observed that ARFIMA (1, 1.2, 1)-FIGARCH (1, 1) outperforms ARFIMA (1, 1.2, 1) and FIGARCH (9, 1) with least AIC.

Table 6 Evaluation Metric Measures of the selected best Models

CPI	ARFIMA (1, 1.2, 1)	2.512241	15.40299	3.924664
CPI	FIGARCH (9, 1)	3.036354	15.99727	3.999659
CPI	ARFIMA (1, 1.2, 1)-FIGARCH (1, 1)	0.1406994	0.7004388	0.8369222

From the results obtained in Table 6 it is observed that ARFIMA (1, 1.2, 1)-FIGARCH (1, 1) outperforms ARFIMA (1, 1.2, 1) and FIGARCH (9, 1) with least MAE, MSE, and RMSE.

Table 7 Coefficients of ARFIMA (1, 1.2, 1)-FIGARCH (1, 1) Model

CPI Time Series				
Parameters	Estimate	Standard error	t-value	Pr(>|t|)
μ	3.213564	0.004980	645.2589	0.000000
ω	0.004996	0.001229	4.0653	0.000048
α_1	0.000000	0.124207	0.0000	1.000000
β_1	0.257503	0.135331	1.9028	0.057071
δ	1.000000	0.000314	3189.1263	0.000000

From the above results obtained in Table 7 it is observed that the expected value of the time series is (3.2136). The constant variance parameter that represents the baseline volatility level is (0.0049). The ARCH parameter is

(0.0000), which measures the impact of first lagged squared error on current volatility. The GARCH parameters are (0.2575) it measures the impact of first lagged volatility on current volatility. The fractional differencing parameter is (1.0000) which measures the long memory component in volatility.

Table 8 Ljung-Box Test of ARFIMA (1, 1.2, 1)-FIGARCH (1, 1)

CPI Time Series		
Lag	Statistic	P-value = 0.4191
Lag 1	0.0178	0.8939
Lag 2	0.1180	0.9072
Lag 4	0.3290	0.9806

From the above results obtained in Table 8 it has been observed that all the probability values are (0.8939, 0.9072, 0.9806) are greater than the alpha value (0.05). Thus, the residuals of the model are approximately normally distributed. There is no significant serial correlation.

Table 9 ARCH-LM of ARFIMA (1, 1.2, 1)-FIGARCH (1, 1)

CPI Time Series				
Lag	Statistic	Shape	Scale	P-value
Lag 3	0.001552	0.500	2.000	0.9686
Lag 5	0.004613	1.440	1.667	0.9998
Lag 7	0.007815	2.315	1.543	1.0000

From the above results obtained in Table 9 it has been observed that all the probability values are (0.9686, 0.9998, 1.0000) are greater than the alpha value (0.05). Thus, the residuals of the model are approximately normally distributed. There is no remaining ARCH-effect in the model.

Table 10 ARFIMA-FIGARCH Model Equation

CPI	$\sigma_t^2 = 0.005 + 0.3\sigma_{t-j}^2 + [1-(1-L)(\sigma_t z_t)_t^2]$

Table 11 Forecast Results

CPI Time Series	
November	0.4630
December	0.5525
January	0.6292
February	0.6974
March	0.7594
April	0.8166

From the results obtained in Table 11 it is observed that the forecasted volatility in November 0.4630 suggests moderate relatively stable market conditions. In December the volatility 0.5525 indicates growing uncertainty, possibly due to holiday season demand and supply chain disruptions. In January the volatility 0.6292 rises reflecting post-holiday market adjustment and potential production planning changes. In February the volatility 0.6970 continued increase in volatility suggests growing market uncertainty influenced by global economic factors and weather events. In March 0.7559 indicates significant price fluctuations, potentially driven by seasonal demand and production changes. In April very high volatility of 0.8166 which suggests extreme market uncertainty, potentially impacting food security and prices.

This study analysed two commodity prices variable (Cereal Prices Index). The study found that the variable has no constant mean and variance that there means and variances change over time. The study found presence of long memory in the variable and long memory is persistent. The study looked at six months ahead forecast for the variable that is from November to April. This study employed hybrid time series model (ARFIMA-FIGARCH) which is good in capturing long memory and heteroscedasticity in economic variables. The study found that the hybrid model outperforms the stand alone models (ARFIMA and FIGARCH) models with least

AIC, MAE, MSE, and RMSE. The forecasted volatility indicates an increasing trend in the variable under study. This result is in consistent with Ngome (2022) and Benbachir (2025) studies. The increasing volatility in each variable has significant implications that include:

- Food security: Rising prices may impact food affordability and accessibility, particularly for vulnerable populations.
- Farmers: increased price risk may influence planting decisions and investment in agricultural production.
- Policymakers: Government may need to intervene to stabilize prices and ensure food security.

Moreover, the increasing volatility of the variable suggests that the variable exhibit long memory, meaning that past shocks and volatility persist over time. This implies that:

- Volatility clustering: periods of high volatility are likely to be followed by more high volatility.
- Slows decay: volatility shocks take time to dissipate, leading to prolonged periods of price instability.

Predictability: The forecasts suggest some predictability in volatility, allowing for informed decision-making.

CONCLUSION

The time series data CPI consist of trend over the long period of time. In addition, the time series (CPI) suffers with fluctuation of mean and variance, these characteristics made them not stationary. Furthermore, the study revealed presence of long memory in the variable under study. Moreover, there is presence of heteroscedasticity in the residuals of ARFIMA (1, 1.2, 1). ARFIMA (1, 1.2, 1)-FIGARCH (1, 1) was the best model with least AIC, MAE, MSE, and RMSE when compared with the ARFIMA and FIGARCH models. There is no significant serial correlation in the residuals, indicating the model reduced volatility persistence. Hence, the study highlights the used of robust model such as ARFIMA-FIGARCH model in proper modelling and forecasting CPI time. Finally, volatility forecasts reveal that CPI volatility increases over time, indicating rising price uncertainty that may adversely affect food affordability and market stability in Nigeria.

People should keep a close eye on dairy market news and trends to stay informed about potential price movements.

i. People should consider implementing risk management strategies, such as hedging or diversification, to mitigate the impact of volatility.
ii. People should develop a contingency plans to address potential price scenarios, including extreme price movements.
iii. People should implement robust risk management strategies to mitigate the impact of volatility on CPI.
iv. People should consider diversifying portfolios to reduce exposure to cereal price fluctuations.

People should continuously monitor market developments and adjust strategies accordingly.

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