



On the New Operations Results Over Intuitionistic Fuzzy Sets

Yusuf A. O.^{1*} & Usman U.²

^{1,2}Department of Mathematics, Federal University Dutsin-Ma. Katsina State. Nigeria.

*Corresponding Author Email:



ABSTRACT

In this Paper, we give a critical study of the development of soft set theory, as an instrument for dealing with uncertainty problems with its various applications and studied the new operations results over Intuitionistic Fuzzy Sets. The new operations on the intuitionistic fuzzy sets over a given universe are presented and some relevant results based on these operations were established.

Keywords:

Soft set,
Fuzzy sets,
Operations.

INTRODUCTION

Soft set is parameterized general mathematical tools which deal with a collection of approximate description of objects. Each approximate has two parts; a predicate and an approximate value set. Soft set is a non standard set. To solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. There are three theories: theory of probability, theory of fuzzy sets, and the interval mathematics which we can consider as mathematical tools for dealing with uncertainties. But all these theories have their own difficulties. In classical mathematics, we construct a mathematical model of an object and define the notion of the exact solution of this model. Usually the mathematical model is too complicated and we cannot find the exact solution so in the second step, we introduce the notion of approximate solution and calculate that solution.

In the soft set theory, we have the opposite approach to this problem. The initial description of the object has an approximate nature, and we do not need to introduce the notion of exact solution the absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. We can use any parameterization we prefer with the help of words and sentences, real number, functions, mappings and so on.

It means that the problem of setting the membership function or any similar problem does not arise in the soft set theory. The Intuitionistic fuzzy set (IFS) introduced by (Atanassov. K, 1999), has become a popular topic of research in the fuzzy set community. There exists a large amount of literature involving IFS theory and some relevant applications.

At the current time, works on the soft set theory are presently in progressing mode. Some works on the intuitionistic fuzzy sets have been carried out by various

researchers, K. Atanassov, (1984, 1986), J. Zhou, and et al. (2011), K.T. Atanossosov, (1995; 2000), P.K. Maji, R. Biswa, A.R. Roy, (2001), J. Zhou, Y.B. Jun, (2010), W. Zeng, H. Li, (2006), L. A. Zadeh, (1965), L. C. Atanassova, (1995).

The aim of this research is to point out some new operation on the intuitionistic fuzzy sets and also study some results relevant to these new operations

MATERIALS AND METHODS

PRELIMINARIES OF SOFT SET THEORY

In this chapter, the fundamentals of soft set theory, operations and its various algebraic structures are presented

A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$ in other words a soft set over U is a parameterized family of subsets of the universe U for $x \in A$, $F(x)$ may be considered as the set of x - approximate elements of the soft set (F, A)

For two soft sets (F, A) and (G, B) over a common universe U , we say that (F, A) is a soft subset of (G, B) if

$A \subseteq B$ and $F(e) \subseteq G(e)$ for all $e \in A$

We write $(F, A) \tilde{\subset} (G, B)$

In case (G, B) is said to be a soft super set of (F, A)

Two soft set (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

Let U be an initial universe, E be a set of parameters, and $A \subseteq E$.

(F, A) is called a relative null soft set (with respect to the parameter set A) denoted by ϕ_A , if $F(a) = \phi$ for all $a \in A$

(G, A) is called a relative whole soft set (with respect to the parameter set A), denoted by U_A if $G(e) = U$ all $e \in A$.

Extended union of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cup_E (G, B) = (H, C)$

Let (F, A) and (G, B) be two soft sets over the same universe U, such that $A \cap B \neq \emptyset$. The restricted union of (F, A) and (G, B) is denoted by $(F, A) \cup_R (G, B)$ and is defined as $(F, A) \cup_R (G, B) = (H, C)$ where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cup G(e)$.

If $A \cap B = \emptyset$, then $(F, A) \cup_R (G, B) = \emptyset$.

The extended intersection of two soft sets (F, A) and (G, B) over a common universe U, is the soft set (H, C) where $C = A \cap B$ and for $e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}$$

We write $(F, A) \cap_E (G, B) = (H, C)$

Let (F, A) and (G, B) be two soft sets over the same universe U such that $A \cap B \neq \emptyset$.

The restricted intersection of (F, A) and (G, B) is denoted by $(F, A) \cap_R (G, B)$ and is defined as $(F, A) \cap_R (G, B) = (H, C)$ where $H(e) = F(e) \cap G(e)$ for all $e \in A \cap B$. If $A \cap B = \emptyset$ then $(F, A) \cap_R (G, B) = \emptyset$

Let (F, A) and (G, B) be two soft set over the same universe U such that $A \cap B \neq \emptyset$. The restricted difference of (F, A) and (G, B) is denoted by $(F, A) \setminus_R (G, B)$, and is defined as $(F, A) \setminus_R (G, B) = (H, C)$, where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) - G(e)$, the difference of the sets F(e) and G(e). If $A \cap B = \emptyset$ then $(F, A) \setminus_R (G, B) = \emptyset$.

The complement of soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \rightarrow \mathcal{P}(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$ for all $\alpha \in A$. clearly, $(F, A)^c = U_A \setminus_R (F, A)$ and $((F, A)^c)^c = (F, A)$.

The class of all value set (F, A) is called value- class of the soft and is denoted by $C_{(F, E)}$. Clearly, $C_{(F, E)} \subseteq \mathcal{P}(U)$.

RESULTS AND DISCUSSION

On the New Operations Results Over Intuitionistic Fuzzy Sets

Definition 1:

Let E be a fixed set. An intuitionistic fuzzy set or IFS A of E is an object having the form: $A = \{\langle x, \mu_A(x), \gamma_A(x) \rangle : x \in E\}$ where the function $\mu_A : E \rightarrow [0, 1]$ and $\gamma_A : E \rightarrow [0, 1]$ define, respectively, the degree of membership and the degree of non-membership of the element of $x \in E$ to the set A, which is a subset of E, and for every element $x \in E$, $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$. So, we have $\phi_A(x) = 1 - \mu_A(x) + \gamma_A(x)$.

Definition 2:

If A and B are two IFSs of the set E, then;

i. $A \subseteq B$ if and only if $\forall x \in E, \mu_A(x) \leq \mu_B(x)$ and

$\gamma_A(x) \geq \gamma_B(x)$

ii. $A = B$ if and only if $\forall x \in E, \mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$

iii. $\Delta A = \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in E\}$

iv. $\nabla A = \{\langle x, 1 - \gamma_A(x), \gamma_A(x) \rangle : x \in E\}$.

Definition 3:

The normalization of an intuitionistic fuzzy set A of the universe U, denoted by $NORM(A)$ simply $N(A)$ is define by $N(A) = \{\langle x, \mu_{N(A)}(x), \gamma_{N(A)}(x) \rangle : x \in U\}$ where

$$\mu_{N(A)}(x) = \frac{\mu_A(x)}{\sup(\mu_A(x))}, \gamma_{N(A)}(x) = \frac{\gamma_A(x) - \inf(\gamma_A(x))}{1 - \inf(\gamma_A(x))}$$

Example 1:

Let the universe $U = \{x_1, x_2, x_3, x_4\}$, and an IFSA = $\{\langle 0.6, 0 \rangle, \langle 0.8, 0 \rangle, \langle 0.5, 0.3 \rangle, \langle 0.4, 0.2 \rangle\}$,

$\sup(\mu_A(x)) = 0.8$, $\inf(\gamma_A(x)) = 0$, thus, we have,

$$\mu_{N(A)}(x_1) = \frac{0.6}{0.8} = 0.75, \quad \mu_{N(A)}(x_2) = \frac{0.8}{0.8} = 1,$$

$$\mu_{N(A)}(x_3) = \frac{0.5}{0.8} = 0.625,$$

$$\mu_{N(A)}(x_4) = \frac{0.4}{0.8} = 0.50$$

$$\gamma_{N(A)}(x_1) = \frac{0-0}{1-0} = 0, \quad \gamma_{N(A)}(x_2) = \frac{0-0}{1-0} = 0,$$

$$\gamma_{N(A)}(x_3) = \frac{0.3-0}{1-0} = 0.3,$$

$$\gamma_{N(A)}(x_4) = \frac{0.2-0}{1-0} = 0.2. \text{ It is obvious that,}$$

$$\mu_{N(A)}(x_1) + \gamma_{N(A)}(x_1) < 1, \quad \mu_{N(A)}(x_2) + \gamma_{N(A)}(x_2) < 1, \quad \mu_{N(A)}(x_3) + \gamma_{N(A)}(x_3) < 1, \quad \text{and} \quad \mu_{N(A)}(x_4) + \gamma_{N(A)}(x_4) < 1.$$

Hence, $N(A)$ is an intuitionistic fuzzy set of the universe U.

Example 2: Let the universe $U = \{x_1, x_2, x_3, x_4\}$, and an IFS $A = \{\langle 0.6, 0 \rangle, \langle 0.8, 0.2 \rangle, \langle 0.7, 0.3 \rangle, \langle 0.4, 0.2 \rangle\}$, then

$\sup(\mu_A(x)) = 0.8$, $\inf(\gamma_A(x)) = 0$, thus, we have,

$$\mu_{N(A)}(x_1) = \frac{0.6}{0.8} = 0.75, \quad \mu_{N(A)}(x_2) = \frac{0.8}{0.8} = 1,$$

$$\mu_{N(A)}(x_3) = \frac{0.7}{0.8} = 0.875,$$

$$\mu_{N(A)}(x_4) = \frac{0.4}{0.8} = 0.50$$

$$\gamma_{N(A)}(x_1) = \frac{0-0}{1-0} = 0, \quad \gamma_{N(A)}(x_2) = \frac{0.2-0}{1-0} = 0.2,$$

$$\gamma_{N(A)}(x_3) = \frac{0.3-0}{1-0} = 0.3,$$

$$\gamma_{N(A)}(x_4) = \frac{0.2-0}{1-0} = 0.2. \text{ It is obvious that,}$$

$$\mu_{N(A)}(x_2) + \gamma_{N(A)}(x_2) = 1.2 > 1, \quad \text{and} \quad \mu_{N(A)}(x_3) + \gamma_{N(A)}(x_3) = 1.175 > 1.$$

Hence, $N(A)$ is not an intuitionistic fuzzy set of the universe U.

However, from the above example, it shows that, the normalization of fuzzy set A of the universe U, denoted by $N(A)$ in example 1, is an intuitionistic fuzzy set, since it satisfies the inequality $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$, and the normalization of fuzzy set A of the universe U, denoted by (A) in example 2 is not an intuitionistic fuzzy set, since $\mu_{N(A)}(x_2) + \gamma_{N(A)}(x_2) = 1.2 > 1$, and also $\mu_{N(A)}(x_3) + \gamma_{N(A)}(x_3) = 1.175 > 1$.

Some new operations on the intuitionistic fuzzy sets:

Theorem 1: For every IFS_s A and B of the set E, the following operations and relations are valid;

- $A \subset B$ if and only if $(\forall x \in E)(\mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x))$;
- $A = B$ if and only if $A \subset B$ and $B \subset A$;
- $\bar{A} = \{\langle x, \gamma_A(x), \mu_A(x) \rangle : x \in E\}$
- $A \cap B = \{\langle x, \min(\mu_A(x), \mu_B(x), \max(\gamma_A(x), \gamma_B(x))) \rangle : x \in E\}$
- $A \cup B = \{\langle x, \max(\mu_A(x), \mu_B(x), \min(\gamma_A(x), \gamma_B(x))) \rangle : x \in E\}$
- $A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \gamma_A(x) \cdot \gamma_B(x) \rangle : x \in E\}$
- $A \cdot B = \{\langle x, \mu_A(x) \cdot \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \cdot \gamma_B(x) \rangle : x \in E\}$

Theorem 2: The operation $\cap$ and $\cup$ are commutative, associative, distributive to the left and to the right among themselves, idempotent, and satisfy the law of De Morgan.

Theorem 3: The operation $+$ and $\cdot$ commutative, associative, and satisfy a law similar to the De Morgan law.

Theorem 4: The operations $+$ and $\cdot$ are distributive to the left and to the right with respect to the operation $\cap$ and $\cup$. Given the set of all intuitionistic fuzzy set IFS's, we shall define two operators over the set of IPS's. They are similar to the operators 'necessity' and 'possibility' defined in some model logic, for every IFS's A.

$$\Delta A = \{\langle x, \mu_A(x) \rangle : x \in E\}$$

$$= \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle : x \in E\},$$

$$\nabla A = \{\langle x, \gamma_A(x) \rangle : x \in E\} = \{\langle x, 1 - \gamma_A(x), \gamma_A(x) \rangle : x \in E\}.$$

Theorem 4.4: For every IFS's

- $\Delta A = \nabla \bar{A}$;
- $\nabla A = \Delta \bar{A}$;

$$\text{iii. } \Delta A \subset A \subset \nabla A;$$

$$\text{iv. } \Delta \Delta A = \Delta A;$$

$$\text{v. } \Delta \nabla A = \Delta A;$$

$$\text{vi. } \nabla \nabla A = \nabla A$$

Theorem 4.5: For every two IFS's A and B;

- $\Delta(A \cup B) = \Delta A \cup \Delta B$;
- $\nabla(A \cup B) = \nabla A \cup \nabla B$

Theorem 4.6: For every two IFS's A and B;

- $\Delta(A \cap B) = \Delta A \cap \Delta B$;
- $\nabla(A \cap B) = \nabla A \cap \nabla B$
- $\Delta(A + B) = \Delta A + \Delta B$;
- $\nabla(A + B) = \nabla A + \nabla B$
- $\Delta(A \cdot B) = \Delta A \cdot \Delta B$;
- $\nabla(A \cdot B) = \nabla A \cdot \nabla B$

Proof (i)

$$\Delta(A \cap B)$$

$$= \Delta\{\langle x, \min(\mu_A(x), \mu_B(x)), \max(\gamma_A(x), \gamma_B(x)) \rangle : x \in E\}$$

$$= \{\langle x, \min(\mu_A(x), \mu_B(x)) \rangle : x \in E\}$$

$$= \{\langle x, \mu_A(x) \rangle : x \in E\} \cap \{\langle x, \mu_B(x) \rangle : x \in E\}$$

$$= \Delta A \cap \Delta B$$

Proof (vi)

$$\nabla(A \cdot B) = \nabla\{\langle x, \mu_A(x) \cdot \mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \cdot \gamma_B(x) \rangle : x \in E\}$$

$$= \{\langle x, 1 - \gamma_A(x) - \gamma_B(x) + \gamma_A(x) \cdot \gamma_B(x) \rangle : x \in E\}$$

$$= \{\langle x, \gamma_A(x) + \gamma_B(x) - \gamma_A(x) \cdot \gamma_B(x) \rangle : x \in E\}$$

$$= \{\langle x, \gamma_A(x) \rangle : x \in E\} + \{\langle x, \gamma_B(x) \rangle : x \in E\}$$

$$= \nabla A + \nabla B.$$

The relations $\subset_\Delta$ and $\subset_\nabla$ are possible between two intuitionistic fuzzy sets if and only if the following holds:

- $A \subset_\Delta B$ if and only if for all x in E , $\mu_A(x) \leq \mu_B(x)$
- $A \subset_\nabla B$ if and only if for all x in E , $\gamma_A(x) \geq \gamma_B(x)$

Theorem 5: For every two IFS's

- $A \subset_\Delta B$ if and only if $\Delta A \subset \Delta B$
- $A \subset_\nabla B$ if and only if $\nabla A \subset \nabla B$

Proof (i):

Let $A \subset_\Delta B$, i.e. $(\forall x \in E)(\mu_A(x) \leq \mu_B(x))$. Then for ΔA and ΔB it follows that $\Delta A \subset \Delta B$. Contrary, if $\Delta A \subset \Delta B$, then $(\forall x \in E)(\mu_A(x) < \mu_B(x))$, i.e., $A \subset_\Delta B$.

Proof (ii):

Let $A \subset_\nabla B$, i.e., $(\forall x \in E)(\gamma_A(x) \geq \gamma_B(x))$. Then for ∇A and ∇B it follows that $\nabla A \supset \nabla B$. Contrary, if $\nabla A \supset \nabla B$, then $(\forall x \in E)(\mu_A(x) > \mu_B(x))$, i.e., $A \subset_\nabla B$.

We introduce a relation " $\sim$ " between A and B, as $A \sim B$ if and only if $(\forall x \in E)(\pi_A(x) \leq \pi_B(x))$.

Theorem 6: For every two IFS's A and B;

- $A \subset_\Delta B$ and $A \subset_\nabla B$ if and only if $A \subset B$;
- if $A \subset_\Delta B$ and $A \sim B$, then $A \subset B$
- if $A \subset_\nabla B$ and $B \sim A$, then $A \subset B$

Proof (i):

If $A \subset_\Delta B$ and $A \subset_\nabla B$ then $(\forall x \in E)(\mu_A(x) \leq \mu_B(x))$ and $(\forall x \in E)(\gamma_A(x) \geq \gamma_B(x))$

Hence, $(\forall x \in E)(\mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x))$. Therefore $A \subset B$. Contrary, if $A \subset B$, then $(\forall x \in E)(\mu_A(x) \leq \mu_B(x) \text{ and } \gamma_A(x) \geq \gamma_B(x))$, i.e., $A \subset_\Delta B$ and $A \subset_\nabla B$

Definition 4:

Let A be a given IFS. We determine for it's the four numbers, $K = \max_{x \in E} \mu_A(x)$, $L = \min_{x \in E} \gamma_A(x)$, $k = \min_{x \in E} \mu_A(x)$, $l = \max_{x \in E} \gamma_A(x)$, and the set $C(A) = \{\langle x, K, L \rangle : x \in E\}$, $I(A) = \{\langle x, k, l \rangle : x \in E\}$ which will be called closure and interior.

Obviously $0 \leq K + L \leq 1$, because if $K = \mu_A(x_1)$ for some $x_1 \in E$ then $0 \leq \mu_A(x_1) + \gamma_A(x_1) \leq 1$, but if $L = \gamma_A(x_1)$, then $x_2 \in E$ will exists such that $L =$

$\gamma_A(x_2) \leq \gamma_A(x_1)$ and therefore $0 \leq K + L \leq 1$.

Analogously $0 \leq k + l \leq 1$. It is therefore follows directly that the following is valid:

Theorem 7: For every two IFS's A and B;

(i) $C(A)$ and $I(A)$ are IFS's;

(ii) $I(A) \subset A \subset C(A)$;

(iii) $C(A \cup B) = C(A) \cup C(B)$;

(iv) $C(C(A)) = C(A)$;

(v) $C(\bar{0}) = \bar{0}$, where $\bar{0} = \{\langle x, 0, 1 \rangle : x \in E\}$.

Proof (iii)

$$\begin{aligned} C(A \cup B) &= C(\{(x, \max(\mu_A(x), \mu_B(x)), \min(\gamma_A(x), \gamma_B(x))) : x \in E\}) \\ &= \{(x, K, L) : x \in E\}, \text{ where} \\ K &= \max_{x \in E} (\max(\mu_A(x), \mu_B(x))) \\ &= \max_{x \in E} (\max(\mu_A(x)), \max(\mu_B(x))) \\ L &= \min_{x \in E} (\min(\gamma_A(x), \gamma_B(x))) \\ &= \min_{x \in E} (\min(\gamma_A(x)), \min(\gamma_B(x))) \end{aligned}$$

Then

$$\begin{aligned} C(A \cup B) &= \left\{ \left\langle x, \max_{x \in E} (\max(\mu_A(x), \mu_B(x))), \min_{x \in E} (\min(\gamma_A(x), \gamma_B(x))) \right\rangle \right\} \\ &= \left\{ \langle x, \max_{x \in E} \mu_A(x), \min_{x \in E} \gamma_A(x) \rangle \right\} \\ &\quad \cup \left\{ \langle x, \max_{x \in E} \mu_B(x), \min_{x \in E} \gamma_B(x) \rangle : x \in E \right\} \\ &= C(A) \cup C(B). \end{aligned}$$

Proof (iv)

$$\begin{aligned} C(C(A)) &= C(\{(x, \max_{x \in E} \mu_A(x), \min_{x \in E} \gamma_A(x)) : x \in E\}) \\ &= \{(x, \max_{x \in E} \mu_A(x), \min_{x \in E} \gamma_A(x)) : x \in E\} \\ &= C(A). \end{aligned}$$

CONCLUSION

In this work, the new operations on the intuitionistic fuzzy sets over a given universe are presented and some relevant results based on these operations were established.

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