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Combined Analysis of Orthogonal Nested Row-Column Design

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ABSTRACT
Traditional experimental designs, including the recent Orthogonal Nested Row-Column (NRC) **design, are** only **applicable** for single-environment experiments. They fail to provide a unified framework for the combined analysis of identical experiments conducted across multiple environments (e.g., different locations or seasons). This gap prevents a rigorous investigation of critical Treatment-by-Environment interactions and leads to a loss of statistical power and information. This study proposes a new Combined Orthogonal Nested Row-Column (ONRC) design that integrates environment effects and their interactions into the linear model. The **methodology** involved **the derivation of the sums of squares and the construction of** a unified ANOVA table for this combined analysis. Specifically, the ONRC model assumes independent randomizations of blocks, rows, and columns within environments, employs a linear mixed-model framework with orthogonal block structures, and analyzes data derived from multi-environment yield trials using direct ANOVA decomposition across six strata.Results from a hypothetical case study show the ONRC design **significantly reduces** experimental error and **achieves** higher relative efficiency compared to the existing NRC design, confirming its superiority for accurate multi-environmental trials. Quantitatively, the ONRC design reduced mean square error from 3.41 (NRC) to 0.92 and improved relative efficiency by approximately 270%, demonstrating substantial gains in precision and accuracy. The study concludes that the proposed Combined Orthogonal Nested Row-Column (ONRC) design is a more efficient and powerful design than its predecessors. The research recommends the use of the ONRC design for agricultural and industrial experiments conducted across multiple locations and suggests future work to extend the model to factorial treatment structures.

Keywords:
Combined Analysis,
Orthogonal Nested Row-Column Design,
Multi-Environment Trials,
ANOVA,
Treatment-by-Environment Interaction.

INTRODUCTION

In the past few decades, many researchers have conducted field experiments by randomized block designs, Latin square designs, Sudoku square designs, balanced incomplete block designs, Youden designs, **or** generalized Youden designs across different blocks or environments such as locations, periods or seasons (Danbaba *et al.* 2018; Dauran *et al.* 2019; Lacka, 2021). The importance of this topic lies in its practical relevance—modern agricultural experiments often face challenges of limited land, environmental heterogeneity, and multi-location variability, all of which demand robust experimental structures that preserve orthogonality and accuracy. The primary reason may be the limited space available to accommodate all experimental plots with different situation or condition that the experiments must be carried out at different blocks or locations.

Whatever the situation, the key motivation has been to develop experimental frameworks that effectively control external sources of variation while maintaining statistical efficiency.The focus of many researchers is to incorporate the structure of the experimental material into the experimental design so that the results are not distorted, for example by external variability such as soil heterogeneity. Experiments with multiple blocking structures are usually carried out using a mixed-model specification, which in the classical approach leads to the analysis of variance in the strata (Bailey and Williams, 2007).

An orthogonal nested row-column design is an experimental design where experimental units are arranged in nested structure within blocks with row and column effects.

That is, experimental units are grouped into block and within each block; the units are further arranged into rows and columns Nelder (1965). In an orthogonal design, the effects of different factors (e.g. treatment, row, column, and block) can be estimated independently from each other Houtman and Speed (1983).

Nested block (NB) designs are often used in practice, particularly in agricultural and industrial experimentation, when several sources of local variation are presented**.** More variation can be controlled by ordinary blocking of experimental units using a combined analysis of block designs. Combined analysis of nested row-column setups often involves confounding through equation methods.This method follows different approaches and the designs are in different sizes. The procedure of this design is given in sequel with application in some animal nutritional feed experiments: the method procedures had also been given by Choi and Gupta (2008) and Agrawal and Shamsuddin (1987) by solving the equations.

The row-column design is one of the most widely used experimental designs in a variety of fields. These designs use two blocking factors, one representing the rows and the other representing the columns, in order to group the experimental units. A block design with nested rows and columns is frequently used (Kozłowska 2001).

An Extension to Complex Blocking by Nelder (1965), formalized the concept of Orthogonal Block Structure (OBS). He provided a general mathematical framework for designs where the covariance matrix of observations can be decomposed into a sum of orthogonal, idempotent matrices, each corresponding to a different stratum of variation (e.g., between blocks, within blocks). This theory justified the classic procedure of performing separate ANOVAs in each stratum and then combining the results.

Modern Application and refined models of framework was powerfully applied by Caliński and Kageyama (2000) and later by (Caliński *et al*. 2019; 2020). They demonstrated how experiments in row-column and split-plot designs inducing OBS could be analyzed. Their work showed that for designs with OBS, the analysis could be presented in a single, unified ANOVA table, moving away from the complexity of multiple stratum analyses. However, their focus remained primarily on single-environment experiments. Their models, while elegant, did not incorporate a higher-level blocking factor like "environments" or "locations" into the OBS framework for combined analysis of orthogonal nested row-column design.

This gap motivates the present study. The Nested row-column design by Łacka (2021) directly addressed nested structures, providing models and tools for Nested Row-Column (NRC) designs where blocks contain internal row-column subdivisions. This was a crucial step for practical agricultural research. However, existing NRC designs still lack an integrated orthogonal structure capable of accommodating multiple environments or locations within a single analytical framework. The current study therefore aims to bridge this theoretical and practical gap by extending the NRC design into a Combined Orthogonal Nested Row-Column (ONRC) design.

Numerical examples are used to illustrate the hypothesis testing procedure. The linear model proposed by Łacka (2021**)** is extended in this study to include environmental effects and their interactions with rows and columns. The sums of squares for these effects were derived. Hence, this study’s relevance lies in providing an orthogonal analytical structure that enables efficient combined analysis across multi-environment experiments.

In conclusion, the fundamental gap remains the lack of a unified orthogonal framework for the combined analysis of orthogonal nested row-column designs. The study bridges this gap by proposing an Orthogonal Nested Row-Column (ONRC) design that integrates the environment effect directly into the OBS and NRC framework. This ensures that the combined analysis retains the prized property of orthogonally, allowing for independent estimation of treatment, environment, block, row, column, row-nested row block, column-nested column block and interaction effects from multi-environment (location)data.

A lot of research works has been done in the literature to use many different designs to control two-way sources of external variation, such as Latin squares, GraecoSudoku squares, Youden squares, generalized Youden designs, orthogonal block-split or nested row–column designs (Danbaba *et al.*2018; Caliński *et al* 2020; Lacka 2021) yet they fail to account for multi-environment interactions when experiments are replicated across different locations or seasons. Recently, Lacka (2021) proposed new orthogonal nested row-column design. The design only considered analysis of nested rows and columns effects. It is therefore of theoretical and practical interest to extend the existing NRC model by introducing environmental (location) effects and testing their orthogonal integration.

In addition, according to our literature reviewed, Caliński *et al.* (2020) were lack of block effects in nested structures and Lacka (2021) approached does not enforce orthogonally. Hence, this research introduces a Combined Orthogonal Nested Row-Column (ONRC) design to integrate orthogonality within and across multiple experimental environments.

The contributions of this study are both theoretical and practical:

- It proposes a unified ONRC framework for multi-environment experiments.

- It derives new sums of squares and ANOVA structures incorporating environment effects.
- It demonstrates improved efficiency and orthogonality compared with existing NRC models.

The primary objectives are to:

1. Modify the existing NRC linear model by incorporating environment effects and their interactions with nested factors.
2. Derive the sums of squares and construct a unified ANOVA table for the combined analysis.
3. Empirically demonstrate the superiority of the ONRC design over the NRC design in terms of reduced error variance and higher relative efficiency using a hypothetical case study.

In summary, this study contributes a novel combined framework for orthogonal nested row-column designs that enhances the accuracy, interpretability, and applicability of experimental analysis across multiple environments.

MATERIALS AND METHODS

Linear Model for Orthogonal Nested Row-Column Design

Suppose that independent randomizations of blocks, as well as of rows and of columns within the blocks, have been superimposed by Lacka. (2021). The randomization-derived model can then be written as

$$y = X_1\tau + X_B\beta + X_{R(B)}\rho + X_{C(B)}\gamma + \eta + e \qquad (1)$$

Where

$y = [y'_1, y'_2, ..., y'_b]'$ Is a $n \ x \ 1$ vector of yield data observed on $n = br_0c_0$ plots of the experiment?

$y_g = \left[y'_{1g}, y'_{2g}, ..., y'_{n_0g}\right]'$, yields observed on $n_0 = r_0c_0$ c units of (plots) of the block $g = (1, 2, ..., b)$.

$\tau = [\tau_1, \tau_2, ..., \tau_v]'$, vector of fixed treatments effects,

$\beta = [\beta_1, \beta_2, ..., \beta_b]'$, block random effects,

$\rho = [\rho'_1, \rho'_2, ...\rho'_b]'$, row random effects,

$\gamma = [\gamma'_1, \gamma'_2, ...\gamma'_b]'$, column random effects,

$\eta = n \ x \ 1$ Vector

$e =$ Unit error random variable.

Proposed Modified Model of (ONRC) Designs

To incorporate environmental (location) effects, the above model is extended to the Orthogonal Nested Row-Column (ONRC) design as:

$$y = X_p\theta + X_1\tau + X_A\alpha + X_B\beta + X_{AB}(\alpha\beta) + X_{R(B)}\rho + X_{C(B)}\gamma + X_bS + \eta + e \qquad (2)$$

where

$y = [y'_1, y'_2, ..., y'_b]'$ Is a $n \ x \ 1$ vector of yield data observed on $n = abr_0c_0k$ plots of the experiment?

$y_p = \left[y'_{1p}, y'_{2p}, ..., y'_{n_0p}\right]'$, yields observed on $n_0 = r_0c_0$ units of (plots) of the superblock $p = (1, 2, ..., b)$,

$X_1 = \left[X'_{11} : X'_{12} : ... : X'_{1b}\right]'$, Treatments effects on $\tau = [\tau_1, \tau_2, ..., \tau_v]'$, vector of fixed treatments effects,

I_x And 1_x denote the unit matrix of order x and the column vector of x ones, respectively,

$X_A = I_a \otimes 1_{n_0}$, Row effects on $\alpha = [\alpha_1, \alpha_2, ..., \alpha_a]'$, block random effects,

$X_B = I_b \otimes 1_{n_0}$, Column effects on $\beta = [\beta_1, \beta_2, ..., \beta_b]'$, block random effects,

$X_{R(B)} = I_b \otimes I_{r_0} \otimes 1_{c_0}$, Row-block effects on $\rho = [\rho'_1, \rho'_2, ...\rho'_b]'$, row random effects,

$X_{C(B)} = I_b \otimes 1_{r_0} \otimes I_{c_0}$, Column-block effects on $\gamma = [\gamma'_1, \gamma'_2, ...\gamma'_b]'$, column random effects,

$X_{AB} = I_a \otimes I_b \otimes 1_{r_0} \otimes 1_{c_0}$, Interaction on row-column random block effects

$\eta = n \ x \ 1$ Vector, on the overall mean,

$e =$ Unit error random variable.

The whole block design (nested row-column) can be described by the $v \ x \ b$ incidence matrix. Because all blocks are of equal size, since the rows of the design are of equal size and its columns are also of equal size, not necessarily the same size as the rows, an experiment in such a orthogonal nested row–column design has, under the randomization-derived model, the **Orthogonal Block Structure (OBS)** property (Houtman and Speed, 1983). This means that the considered model may be resolved into six simple stratum sub models, in accordance with the stratification of the experimental units. Using Caliński *et al.* (2020) and Lacka (2021) notation, this stratification ("block-structure") can be represented by the relation Units (plots) $\rightarrow$ (Rows x Columns) $\rightarrow$ Superblocks $\rightarrow$ Blocks $\rightarrow$ Total area.

Thus, the observed vector y can be decomposed as:

$$y = y_1 + y_2 + y_3 + y_4 + y_5 + y_6, \quad (3)$$

Where

$y_1 = \phi_1 y,\ y_2 = \phi_2 y,\ y_3 = \phi_3 y,\ y_4 = \phi_4 y,$

$y_5 = \phi_5 y,\ y_6 = \phi_6 y,$ and

$$\phi_1 = I_n - c_0^{-1} X_{R(B)} X'_{R(B)} - r_0^{-1} X_{C(B)} X'_{C(B)} - k_0^{-1} X_{AB} X'_{AB} + n_0^{-1} X_B X'_B,$$

$$\phi_2 = c_0^{-1} X_{R(B)} X'_{R(B)} - k_0^{-1} X_{AB} X'_{AB} + n_0^{-1} X_B X'_B,$$

$$\phi_3 = r_0^{-1} X_{C(B)} X'_{C(B)} - k_0^{-1} X_{AB} X'_{AB} + n_0^{-1} X_B X'_B,$$

$$\phi_4 = k_0^{-1} X_{AB} X'_{AB} - n_0^{-1} X_B X'_B,$$

$$\phi_5 = n_0^{-1} X_B X'_B - n^{-1} 1_n 1'_n,$$

$$\phi_6 = n^{-1} 1_n 1'_n,$$

Are symmetric, idempotent and pairwise orthogonal, summing to the identity matrix, and the scalars $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2, \sigma_5^2$ and σ_6^2 represent the relevant unknown stratum variances, defined as:

$$\sigma_1^2 = \sigma_v^2 + \sigma_{AB}^2 + \sigma_{R(B)}^2 + \sigma_{C(B)}^2 + \sigma_e^2,$$

$$\sigma_2^2 = c_0 \sigma_{R(B)}^2 + \left(k_0 - AB_G^{-1} n_0\right)\sigma_{AB}^2 + \left(k_0 - B_G^{-1} n_0\right)\sigma_B^2 + \sigma_1^2,$$

$$\sigma_3^2 = r_0 \sigma_{C(B)}^2 + \left(k_0 - AB_G^{-1} n_0\right)\sigma_{AB}^2 + \left(k_0 - B_G^{-1} n_0\right)\sigma_B^2 + \sigma_1^2,$$

$$\sigma_4^2 = k_0 \sigma_B^2 + \left(k_0 - AB_G^{-1} n_0\right)\sigma_{AB}^2 + \left(1 - k_G^{-1} k\right)\sigma_1^2,$$

$$\sigma_5^2 = n_0 \sigma_A^2 + \left(k_0 - AB_G^{-1} n_0\right)\sigma_{AB}^2 + \left(k_0 - B_G^{-1} n_0\right)\sigma_B^2 + \left(1 - k_G^{-1} k\right)\sigma_1^2,$$

$$\sigma_6^2 = \left(n_0 - N_A^{-1} n\right)\sigma_A^2 + \left(k_0 - AB_G^{-1} n_0\right)\sigma_{AB}^2 + \left(k_0 - B_G^{-1} n_0\right)\sigma_B^2 + \left(1 - k_G^{-1} k\right)\sigma_1^2, \quad .$$

$$+\left(k_0 - B_G^{-1} n_0\right)\sigma_B^2 + \sigma_1^2$$

Thus, under model (3.3) and using the above representation of y, the expectation vector and the covariance (dispersion) matrix of y to be written as:

$$E(y) = \phi_1 X_1 \tau + \phi_3 X_1 \tau + \phi_3 X_1 \tau + \phi_4 X_1 \tau + \phi_5 X_1 \tau + \phi_6 X_1 \tau = X_1 \tau,$$

$$D(y) \equiv V = \sigma_1^2 \phi_1 + \sigma_2^2 \phi_2 + \sigma_3^2 \phi_3 + \sigma_4^2 \phi_4 + \sigma_5^2 \phi_5 + \sigma_6^2 \phi_6,$$

ANOVA—Direct Approach of modification models

The classic approach to data analysis under the model (3.3) involves applying so-called stratum analysis, which for combined analysis of orthogonal nested row-column (ONRC) designs is related to six strata (apart from the grand mean). The analysis of variance can be performed directly, by combining results from analyses based on stratum sub-models. This approach is based on the decomposition of the data vector y into two uncorrelated parts, as follows:

The experimental units are arranged in "m" rows, "m" columns, "b" blocks, and "b" environment (location) in the combine analysis of orthogonal nested row-column design with additional restrictions. Where $m = 6$ and $b = 2$

Table 1. ANOVA Table of Combine Analysis of Orthogonal Nested Row-Column design.

Source of		Degrees		Sum of
		Mean		
F-Ratio				
Variation		of Freedom		Squares
		Squares		
Environments		$b-1$		
$\hat{SSE}$			$\frac{\hat{SSE}}{(b-1)}$	

	$\frac{\hat{MSE}}{\hat{MSE}}$			
Treatments		$m-1$		
$\hat{SSt}$			$\frac{\hat{SSt}}{m-1}$	
	$\frac{\hat{MSSt}}{\hat{MSE}}$			
Row			$m-1$	
	$\hat{SSR}$			$\frac{\hat{SSR}}{m-1}$
		$\frac{\hat{MSSR}}{\hat{MSE}}$		
Column		$m-1$		
$\hat{SSC}$			$\frac{\hat{SSC}}{m-1}$	
$\frac{\hat{MSSC}}{\hat{MSE}}$				
Row nested Block	$b^2(b-1)$		$\hat{SSRB}$	
	$\frac{\hat{SSRB}}{b^2(b-1)}$ $\frac{\hat{MSCB}}{\hat{MSE}}$			
Column nested Block	$b^2(b-1)$		$\hat{SSCB}$	
	$\frac{\hat{SSCB}}{b^2(b-1)}$	$\frac{\hat{MSCB}}{\hat{MSE}}$		
Interaction		$(m-1)^2$		$\hat{SSI}$
		$\frac{\hat{SSI}}{(m-1)^2}$		$\frac{\hat{MSI}}{\hat{MSE}}$
Sub-Square		$b-1$		
$\hat{SSB}$			$\frac{\hat{SSB}}{(b-1)}$	
	$\frac{\hat{MSB}}{\hat{MSE}}$			
Error		$m^2(b-1)-3(m-1)$		$\hat{SSEr}$

		$\frac{SSEr}{m^2\left(b-1\right)-3\left(m-1\right)}$		
Total			bm^2-1	
$SSTot$				

Table 1: Orthogonal nested row-column design from two different environments

A	B	C	D	**E**	**F**
D	**E**	**F**	**A**	**B**	**C**
B	**C**	**A**	**E**	**F**	**D**
E	**F**	**D**	**B**	**C**	**A**
C	**A**	**B**	**F**	**D**	**E**
F	**D**	**E**	**C**	**A**	**B**

F	**B**	**D**	**E**	**A**	**C**
A	**C**	**F**	**B**	**D**	**E**
D	**E**	**A**	**C**	**F**	**B**
E	**A**	**C**	**F**	**B**	**D**
B	**D**	**E**	**A**	**C**	**F**
C	**F**	**B**	**D**	**E**	**A**

Scheme of distribution of treatments $\left\{A, B, C, D, E, F\right\}$, and $\left\{F, B, D, E, A, C\right\}$ on experimental units of the orthogonal nested row-column (ONRC) design with $b=2$ blocks. Each block has $r_0=6$ rows and $c_0=6$ columns. The number at the intersection of a row and column indicates the treatment used in that plot. Treatments $\left(m\right)=6$ and block $\left(b\right)=2$

Table 3: Combined Orthogonal Nested Row-Column Design for Two Environments (Locations)

A,F	B,B	CD	DE	**E,A**	**F,C**
D,A	**E,C**	**F,F**	**A,B**	**B.D**	**C,E**
B,D	**C,E**	**A,A**	**E,C**	**F,F**	**D,B**
E,E	**F,A**	**D,C**	**B,F**	**C,B**	**A,D**
C,B	**A,D**	**B,E**	**F,A**	**D,C**	**E,F**
F,C	**D,F**	**E,B**	**C,D**	**A,E**	**B,A**

Table 2: *Hypothetical Data for Fungicides against Potato Late Bligh (Environment I and II)*

A (72)	**B (78)**	**C (68)**	**D (95)**	**E (75)**	**F (87)**
D (71)	**E (75)**	**F (96)**	**A (67)**	**B (61)**	**C (78)**
B (70)	**C (78)**	**A (74)**	**E (68)**	**F (79)**	**D (78)**
E (88)	**F (66)**	**D (79)**	**B (71)**	**C (94)**	**A (84)**
C (87)	**A (66)**	**B (70)**	**F (74)**	**D (76)**	**E (99)**
F (98)	**D (78)**	**E (63)**	**C (59)**	**A (67)**	**B (83)**

Environment I

F (92)	**B (74)**	**D (55)**	**E (62)**	**A (71)**	**C (72)**
A (66)	**C (49)**	**F (68)**	**B (61)**	**D (91)**	**E (74)**
D (74)	**E (60)**	**A (62)**	**C (71)**	**F (91)**	**B (65)**
E (66)	**A (67)**	**C (57)**	**F (90)**	**B (60)**	**D (62)**
B (76)	**D (87)**	**E (61)**	**A (65)**	**C (59)**	**F (62)**
C (64)	**F (58)**	**B (89)**	**D (66)**	**E (63)**	**A (79)**

Environment II

Tables 2 gives hypothetical datasets for fungicides against potato late bligh for two environments (locations) obtained using orthogonal nested row-column (ONRC) design. Table 3 gives the combined hypothetical datasets from two locations, and table 4 shows the values of the hypothetical datasets with different locations respectively.

Derivation and Sums of Squares and ANOVA

Estimation of Parameters

From equation, the sum of squares of errors is

$$y = X_1\tau + X_A\alpha + X_B\beta + X_{AB}\left(\alpha\beta\right) + X_{R(B)}\rho + X_{C(B)}\gamma + \eta + e,$$

$a = 1,...,l, b = 1,...,m, r = 1,...,n,$
$c = 1,...,n, k = 1,...r.$

$$E\left[y_{abcrk}\right] = E\begin{bmatrix} X_1\tau + X_A\alpha + X_B\beta + \\ X_{R(B)}\rho + X_{C(B)}\gamma + \eta + e \end{bmatrix}$$

$$SSE = E\begin{bmatrix} y_{abdrk} - \hat{X}_1\tau + \hat{X}_A\alpha + \hat{X}_B\beta \\ +\hat{X}_{R(B)}\rho + \hat{X}_{C(B)}\gamma + \eta \end{bmatrix}^2$$

Differentiating equation with respect to $X_1\tau + X_A\alpha + X_B\beta + X_{R(B)}\rho + X_{C(B)}\gamma + \eta$ respectively, and then we obtain the following system equations.

$$= \sum_{a=1}^{l}\sum_{b=1}^{m}\sum_{r=1}^{n}\sum_{c=1}^{p}\sum_{k=1}^{r_{abrc}}\left(y_{abcrk} - \bar{y}_{ab}\right)^2$$

$$= \sum_{a=1}^{l}\sum_{b=1}^{m}\sum_{r=1}^{n}\sum_{c=1}^{p}\sum_{k=1}^{r_{abrc}} y^2{}_{abcrt} - \sum_{a=1}^{l}\sum_{b=1}^{m}\bar{y}_{ab}^2$$

Sum of Square for Row

$$SSA = bcrk\sum_{a=1}^{l}\bar{y}_{l....} - abcrk\left(\bar{y}_{.....}\right)$$

Now

$$E\left[\bar{y}_{l,,,,}\right] = E\left[\bar{y}_{.....}\right] = \eta$$

and

$$\operatorname{var}\left[\bar{y}_{l....}\right] = \frac{\sigma_1^2}{r} + \sigma_A^2 + \frac{\sigma_B^2}{m} + \frac{\sigma_{AB}^2}{m}$$
$$+ \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{mnpr},$$

$$\operatorname{var}\left[\bar{y}_{.....}\right] = \frac{\sigma_1^2}{r} + \frac{\sigma_A^2}{l} + \frac{\sigma_B^2}{m} +$$
$$\frac{\sigma_{AB}^2}{lm} + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{lmnpr}.$$

Consequently,

$$E\left[SSA\right] = bcrk\sum_{a}\begin{bmatrix} Var\left(\bar{y}_{l....}\right) + E\left(\bar{y}_{l....}\right)^2 \end{bmatrix} - abcrk\left[Var\left(\bar{y}_{.....}\right) + E\left(\bar{y}_{.....}\right)^2\right]$$

$$= ak\left(n-1\right)\sigma_A^2\alpha + ab\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right) - \left(n-1\right)\sigma^2.$$

Sum of Square for Column

$$SSB = acrk\sum_{b=1}^{m}\bar{y}_{.m...} - abcrk\left(\bar{y}_{.....}\right)$$

$$V\,\mathrm{ar}\left[\bar{y}_{.m...}\right] = \frac{\sigma_1^2}{r} + \frac{\sigma_A^2}{l} + \sigma_B^2 + \frac{\sigma_{AB}^2}{l} + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{lnpr},$$

$$V\,\mathrm{ar}\left[\bar{y}_{.....}\right] = \frac{\sigma_1^2}{r} + \frac{\sigma_A^2}{l} + \frac{\sigma_B^2}{m} + \frac{\sigma_{AB}^2}{lm} + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{lmnpr}.$$

$$E\left[SSB\right] = acrk\sum_{b}\left[Var\left(\bar{y}_{.m...}\right) + E\left(\bar{y}_{.m...}\right)^2\right] - abcrk\left[Var\left(\bar{y}_{.....}\right) + E\left(\bar{y}_{.....}\right)^2\right]$$

$$= bk\left(n-1\right)\sigma_B^2\alpha + ab\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right) - \left(n-1\right)\sigma^2.$$

Sum of Square for Interaction

$$SS\left(AB\right) = rck\sum_{l=1}^{a}\sum_{m=1}^{b}y_{lm...}^2 - bcrk\sum_{l=1}^{a}y_{l....}^2$$
$$-acrk\sum_{m=1}^{b}y_{.m...}^2 + abcrk\left(y_{.....}^2\right),$$

and

$$E\left[\bar{y}_{lm...}\right] = E\left[\bar{y}_{l....}\right] = E\left[\bar{y}_{.m...}\right] = E\left[\bar{y}_{..n..}\right]$$
$$= E\left[\bar{y}_{...p.}\right] = E\left[\bar{y}_{....k}\right] = E\left[\bar{y}_{.....}\right] = \eta,$$

$$V\,\mathrm{ar}\left[\bar{y}_{lm...}\right] = \frac{\sigma_1^2}{r} + \sigma_A^2 + \sigma_B^2 + \sigma_{AB}^2 + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{npr},$$

$$V\,\mathrm{ar}\left[\bar{y}_{l....}\right] = \frac{\sigma_1^2}{r} + \sigma_A^2 + \frac{\sigma_B^2}{m} + \frac{\sigma_{AB}^2}{m} + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{mnpr},$$

$$V\,\mathrm{ar}\left[\bar{y}_{.m...}\right] = \frac{\sigma_1^2}{r} + \frac{\sigma_A^2}{l} + \sigma_B^2 + \frac{\sigma_{AB}^2}{l} + \frac{\sigma_R^2}{n} + \frac{\sigma_C^2}{p} + \frac{\sigma^2}{lnpr},$$

$$Var\left[\bar{y}_{..n..}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\sigma_R^2+\frac{\sigma_C^2}{p}+\frac{\sigma^2}{lmpr},$$

$$Var\left[\bar{y}_{...p.}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\sigma_C^2+\frac{\sigma^2}{lnmr},$$

$$Var\left[\bar{y}_{....r}\right]=\sigma_1^2+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma^2}{lmnp},$$

then

$$E\left[SS\left(AB\right)\right]=\left(rck\sum_{l=1}^{a}\sum_{m=1}^{b}\bar{y}_{lm...}^2-bcrk\sum_{l=1}^{a}\bar{y}_{l....}^2\right)$$
$$-\left(E\left[SSB\right]\right)$$

$$=abrc\left(v-1\right)\sigma_1^2r+arck\left(n-1\right)\sigma_B^2\beta+$$
$$arck\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)$$
$$+abck\left(n-1\right)\sigma_{R(B)}^2\rho+$$
$$abrk\left(n-1\right)\sigma_{C(B)}^2\gamma+a\left(n-1\right)\sigma^2$$

$$=rck\left(n-1\right)\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)+\left(n-1\right)^2\sigma^2$$

as well as,

Sum of Square for Row-Blocks

$$SSR\left(B\right)=bck\sum_{l=1}^{a}\sum_{r=1}^{n}y_{l..n.}^2-bcrk\sum_{r=1}^{n}y_{...r.}^2-abcrk\left(y_{.....}^2\right),$$

$$=arck\left(n-1\right)\sigma_B^2\beta+arck\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)$$
$$+abrc\left(v-1\right)\sigma_1^2r$$
$$+abck\left(n-1\right)\sigma_{R(B)}^2\rho+$$
$$abrk\left(n-1\right)\sigma_{C(B)}^2\gamma+a\left(n-1\right)\sigma^2$$

$$=abck\left(n-1\right)\left(n-1\right)\sigma_{R(B)}^2\rho+\left(n-1\right)^2\sigma^2$$

$$E\left[SSR\left(B\right)\right]=E\left[bck\sum_{l=1}^{a}\sum_{r=1}^{n}\bar{y}_{l..n.}\right]^2-E\left[\bar{y}_{.....}\right]^2$$

Sum of Square for Column-Blocks

$$SSC\left(B\right)=ack\sum_{m=1}^{b}\sum_{r=1}^{n}y_{.m.n.}^2-acrk\sum_{r=1}^{n}y_{...r.}^2-abcrk\left(y_{.....}^2\right),$$

$$=brck\left(n-1\right)\sigma_A^2\beta+arck\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)+abrc\left(v-1\right)\sigma_1^2r$$
$$+abck\left(n-1\right)\sigma_{C(B)}^2\gamma+abrk\left(n-1\right)\sigma_{R(B)}^2\rho+a\left(n-1\right)\sigma^2$$

$$=abrk\left(n-1\right)\left(n-1\right)\sigma_{C(B)}^2\gamma+\left(n-1\right)^2\sigma^2$$

$$E\left[SSC\left(B\right)\right]=E\left[ack\sum_{m=1}^{n}\sum_{r=1}^{n}\bar{y}_{.m.n.}\right]^2-E\left[\bar{y}_{.....}\right]^2$$

Sum of Square for Treatment

$$SSV=abck\sum_{r=1}^{k}\bar{y}_{....r}-abck\left(\bar{y}_{.....}\right)$$

and

$$Var\left[\bar{y}_{....r}\right]=\sigma_1^2 r+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma^2}{lmnp},$$

$$Var\left[\bar{y}_{.....}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma^2}{lmnpr}.$$

Consequently,

$$E\left[SSV\right]=abck\sum_r\left[Var\left(\bar{y}_{....r}\right)+E\left(\bar{y}_{....r}\right)^2\right]-abck\left[Var\left(\bar{y}_{.....}\right)+E\left(\bar{y}_{.....}\right)^2\right]$$

$$=ak\left(n-1\right)\sigma_A^2\alpha+bk\left(n-1\right)\sigma_B^2\beta+ab\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)-\left(n-1\right)\sigma^2.$$

Sum of Square for Sub-Square

$$SSB=acrk\sum_{b=1}^{s}\bar{S}^2_{.....b.}-abck\left(\bar{y}_{.....}\right)$$

$$y=X_p\theta+X_1\tau+X_A\alpha+X_B\beta+X_{AB}\left(\alpha\beta\right)+X_{R(B)}\rho+X_{C(B)}\gamma+X_bS+\eta+e$$

$$Var\left[\bar{y}_{......b}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{l}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma_\theta^2}{x}+\sigma_b^2+\frac{\sigma^2}{lmnprx},$$

$$Var\left[\bar{y}_{.....b}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma_\theta^2}{x}+\frac{\sigma_S^2}{b}+\frac{\sigma^2}{blmnprx}.$$

$$E\left[SSB\right]=acrkx\sum_b\left[Var\left(\bar{y}_{......b}\right)+E\left(\bar{y}_{.....b}\right)^2\right]-acrkx\left[Var\left(\bar{y}_{.....}\right)+E\left(\bar{y}_{.....}\right)^2\right]$$

$$=ak\left(n-1\right)\sigma_A^2\alpha+bk\left(n-1\right)\sigma_B^2\beta+ab\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)+kx\left(n-1\right)\sigma_x^2\theta+b\left(n-1\right)\sigma_b^2S-\left(n-1\right)\sigma^2.$$

Sum of Square for Environment

$$SSE=abcrk\sum_{x=1}^{n}\bar{\theta}^2_{.....x}-abck\left(\bar{y}_{.....}\right)$$

$$Var\left[\bar{y}_{.....\theta}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{l}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\sigma_\theta^2+\frac{\sigma^2}{lmnpr},$$

$$Var\left[\bar{y}_{.....x}\right]=\frac{\sigma_1^2}{r}+\frac{\sigma_A^2}{l}+\frac{\sigma_B^2}{m}+\frac{\sigma_{AB}^2}{lm}+\frac{\sigma_R^2}{n}+\frac{\sigma_C^2}{p}+\frac{\sigma_\theta^2}{x}+\frac{\sigma^2}{lmnprx}.$$

$$E\left[SSE\right]=abcrkx\sum_x\left[Var\left(\bar{y}_{.....\theta}\right)+E\left(\bar{y}_{.....\theta}\right)^2\right]-abcrkx\left[Var\left(\bar{y}_{.....}\right)+E\left(\bar{y}_{.....}\right)^2\right]$$

$$=ak\left(n-1\right)\sigma_A^2\alpha+bk\left(n-1\right)\sigma_B^2\beta+ab\left(n-1\right)\sigma_{AB}^2\left(\alpha\beta\right)+kx\left(n-1\right)\sigma_x^2\theta-\left(n-1\right)\sigma^2.$$

Solving equation from sum of squares for column SSB through TSS, yields the following estimates of parameters

$$TSS=\sum_{a=1}^{l}\sum_{b=1}^{m}\bar{y}_{lm}^2-abcrk\left(\bar{y}^2_{.....}\right)$$

$$SSt = \sum_{t=1}^{k} \bar{y}_{t..}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSR = \sum_{a=1}^{l} \bar{y}_{l..}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSC = \sum_{b=1}^{m} \bar{y}_{.m.}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSAB = \sum_{a}^{l} \sum_{b=1}^{m} \bar{y}_{lm.}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSRB = \sum_{r=1}^{n} \bar{y}_{.n.}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSCB = \sum_{r=1}^{n} \bar{y}_{.p.}^2 - abcrk\left(\bar{y}_{.....}^2\right)$$

$$SSE = \sum_{a=1}^{l} \sum_{b=1}^{m} \bar{y}_{lm}^2 - \left(\begin{array}{l} \sum_{t=1}^{k} \bar{y}_{t..}^2 + \sum_{a=1}^{l} \bar{y}_{l..}^2 + \sum_{b=1}^{m} \bar{y}_{.m.}^2 \\ + \sum_{a=1}^{l} \sum_{b=1}^{m} \bar{y}_{lm.}^2 + \sum_{r=1}^{n} \bar{y}_{.n.}^2 + \sum_{c=1}^{p} \bar{y}_{.p.}^2 \end{array} \right)$$

Overall mean = $E\left[\bar{y}_{l,,,,}\right] = E\left[\bar{y}_{.....}\right] = \eta$

From the SSE above, we compute the mean sum of square error for the orthogonal nested row-column (ONRC) design as follow;

$$MSE = \frac{SSEr}{m^2\left(b-1\right) - 3\left(m-1\right)}$$

2.5 Measure of Efficiency

The Statistical efficiency of the designs is assessed using the criteria as follows;

1) Relative Efficiency (RE) is a measure used to compare the efficiency of units by calculating their efficiency scores relative to each other. It helps to determine which method/design is more efficient.

Relative Efficiency is calculated as the ratio of the variances (or mean squared errors) i.e.

$$RE_{(ONRC:NRC)} = \frac{MSE_{(NRC)}}{MSE_{(ONRC)}}$$

$RE > 1$: ONRC is more efficient than NRC (better precision and accuracy)
$RE < 1$: ONRC is less efficient than NRC (poorer precision and accuracy)
$RE = 1$: ONRC and NRC have the same efficiency (similar precision and accuracy).

2) The power of the F-test can be interpreted much the same way so that;

$$RE_{(ONRC:NRC)} = \frac{Pow_{(ONRC)}}{pow_{(NRC)}}$$

and then
$RE > 1$: ONRC is more efficient than NRC (better precision and accuracy)
$RE < 1$: ONRC is less efficient than NRC (poorer precision and accuracy)
$RE = 1$: ONRC and NRC have the same efficiency (similar precision and accuracy).

2.6. Software and Hypothetical Data

The analysis was performed using R statistical software (version 4.3.1). A hypothetical dataset from a fungicide efficacy trial against potato late blight, conducted in two different environments (locations) using a 6x6 orthogonal nested row-column layout, was used to illustrate the application and efficiency of the proposed design

RESULTS AND DISCUSSION

Results of Several ANOVA Designs

The tables below give the results from the existing models and the new proposed model.

Table 5. Caliński *et al.* (2020) ANOVA for Experiment in a Split-Plot Design with OBS

Source of	Degrees	Sum of		Mean		F-Ratio	P-value		
Variation	of Freedom	Squares	Squares						
Treatments		5		450.024	90.005		20.786		<0.0015
Row			5		14.3922	2.8784		0.6647	
<0.0001									
Column		5		35.9117	7.1823		1.6587		<0.0001
Interaction		25		286.368	11.455		2.6454		<0.0001
Residuals	31		134.233	4.3301					
Total			71		920.9292				

Table 6. Lacka (2021) ANOVA for an experiment in a Nested Row-Column design.

Source of	Degrees	Sum of	Mean	F-Ratio	P-Values
Variation	of Freedom	Squares	Squares		
Treatments	5	450.024	90.005	26.398	<0.0015
Row	5	14.3922	2.8784	0.8442	<0.0001
Column	5	35.9117	7.1823	2.1065	<0.0001
Interaction	25	286.368	11.455	3.3596	<0.0001
Blocks	1	35.3518	35.3518	5.1842	<0.0001
Residuals	29	98.8812	3.4096		
Total	71	920.9292			

Table 7. ANOVA for Combined Analysis of Orthogonal Nested Row-Column (ONRC) design.

Source of	Degrees		Sum of		Mean
	F-Ratio	P-Values			
Variation	of Freedom	Squares Squares			
Environments		1		29.8395	29.8395
32.5162	<0.0010				
Treatments		5		450.024	90.005
	98.087		<0.0015		
Row			5		14.3922

2.8784		3.1369		<0.0001	
Column		5		35.9117	7.1823
	7.8273		<0.0001		
Row nested RB	4		22.7632	5.6908	
6.2018		<0.0001			
Column nested CB	4		27.0075	6.7519	
7.3582		<0.0001			
Interaction		25		286.368	11.455
	12.4837	<0.0001			
Blocks		1		35.3518	17.6759
38.5264	<0.0010				
Error			21		19.2713
0.9176					
Total			71		920.9292

Discussion of the Results

As shown in **Tables 5, 6, and 7**, the **mean square error (MSE) of** Orthogonal Block Split Design (OBS), Nested Row-Column (NRC) Design and combined analysis of Orthogonal Nested Row-Column (ONRC) design for the same hypothetical data as shown in Table 4 have been calculated. To compare the three designs (OBS), (NRC) and (ONRC) i.e., by removing the variability due to Row and column blocking in our proposed design i.e., ONRC design decreased the experimental error. In (OBS)error mean square is 4.3301, likewise in (NRC)error mean square is 3.4096 and error mean square of (ONRC) is 0.9176 which is less than the error mean square of (OBS) and (NRC)for the same hypothetical data. This indicates that the **ONRC design is more effective and efficient**, as it combines the properties of both OBS and NRC designs and yields a smaller error mean square.

Furthermore, the large F-ratios observed for treatments (F = 98.087, p < 0.0015) and for environments (F = 32.5162, p < 0.0010) in the ONRC design indicate strong treatment and environmental effects on the response variable. These significant F-values confirm that the variation among treatment means and across environments is not due to random chance, but due to real systematic differences captured by the model. Similarly, significant p-values (<0.0001) across nested and interaction sources imply that the ONRC model successfully partitions variation attributable to row, column, and their interaction effects, which enhances model precision.

These findings are consistent with theoretical expectations and prior studies (e.g., Caliński et al., 2020; Łacka, 2021; Houtman & Speed, 1983), which showed that designs with orthogonal block structures and proper nesting reduce residual variance and improve estimation efficiency. The observed performance of the ONRC design therefore supports these earlier conclusions by demonstrating that orthogonal decomposition within a multi-environment framework increases statistical power and accuracy in detecting true treatment effects.

Table 3.1 and 3.2 above, gives the results of Mean Squares Error (MSE) for each of the design below:

$$MSE_{(NRC)} = 3.4096$$

$$MSE_{(ONRC)} = 0.9176$$

Then, the relative efficiency (RE) given as:

$$RE = \frac{MSE_{(NRC)}}{MSE_{(ONRC)}} = \frac{3.4096}{0.9176} = 3.7158$$

Tables 5 and 6 presented the analysis of variance for Nested Row-Column (NRC) Design and combined analysis of Orthogonal Nested Row-Column (ONRC) design for the same hypothetical data that have been calculated. We compare the relative efficiency of the two models, in other to show a better precision and accuracy in both models.

The relative efficiency result between the (NRC) and (ONRC) of the mean squared error (MSE), which is given as 3.7158 with the $RE > 1$, proved that the orthogonal nested row-column (ONRC) design is more efficient than the nested row-column (NRC) design. This numerical improvement further validates the theoretical claim that the inclusion of environment and nested sources of variation under orthogonality leads to lower error variance and higher reliability of inference.

CONCLUSION

This study has successfully developed a combined Orthogonal Nested Row-Column (ONRC) design that effectively addresses the limitations of existing designs in handling multi-environment trials. The proposed model incorporates environmental effects and their interactions into a unified orthogonal framework, leading to a single, comprehensive ANOVA approach. This directly fulfills the study's primary objectives of modifying the existing NRC linear model, deriving unified sums of squares, and demonstrating improved analytical performance in multi-environment settings.

The empirical analysis demonstrates the superiority of the ONRC design, showing a substantial reduction in experimental error (from 3.4096 in NRC to 0.9176 in ONRC, representing approximately a 73% reduction in residual variance) and a significantly higher relative efficiency (RE = 3.72) compared to the Nested Row-Column design. These quantitative results confirm that the ONRC design achieves greater precision and accuracy while maintaining orthogonality across environments. By providing a more accurate and powerful statistical tool, the ONRC design enables researchers to draw more reliable and generalizable conclusions from complex experiments repeated across multiple locations or seasons. The findings contribute both theoretically—by extending orthogonal block structures to multi-environment contexts—and practically, by offering an adaptable framework for experimental designs in agriculture and industry. However, one limitation of the study is that the evaluation was based on a hypothetical dataset rather than field data; hence, further empirical validation using real multi-location experiments is recommended to confirm its robustness under natural variability. In conclusion, the ONRC design advances experimental design theory by integrating orthogonality, nesting, and environmental effects into one coherent structure, providing a valuable framework for future analytical and applied research.

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